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An energetic criterion for a micro-crack of finite length initiated in orthotropic bi-material notches

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ABSTRACT

The singular stress field at the tip of a bi-material orthotropic notch may cause the initiation of micro-cracks into one of its material part. Assuming the plane elasticity of the anisotropic media, the Lekhnitskii–Eshelby–Stroh formalism is used to express the characteristics of the singular stress field at the notch tip, the auxiliary solution necessary for the calculation of the contour-independent integrals and consequently generalized stress intensity factors. The matched asymptotic expansion analysis is used to evaluate the fracture energy release rate which allows one to assess the potential direction of a crack and establish the fracture criterion based on the theory of the finite fracture mechanics.

1. Introduction

Joints of different materials, such as layered composite materials, fiber reinforced ceramics or constructions with protective surface layers may occur in practical engineering structures. They make it possible to achieve properties which could not be attained by means of homogeneous materials. On the other hand, most homogeneous crystalline materials which show elastic isotropy on a macro-scale are anisotropic on the scale of individual grains due to the varying orientation of crystal directions. Hence from the micro-scale point of view, grains may form multi-material wedges. These examples show that the investigation of notches as generalized stress singularities and possible crack initiators may play an important role in practical engineering problems as well as in the material design.

In the case of composite materials the stress field in the close vicinity of multi-material joints has a singular character and besides the geometry of the notch it is governed by the overall anisotropic material response. The degree of anisotropy of many advanced materials is lower than in the general anisotropic materials [1]. These kinds of materials possess one or more symmetry planes, e.g. orthotropic materials with three planes of symmetry, which form an important class of degenerate anisotropic materials. For these materials, the case of anti-plane and in-plane strains can be decoupled and this feature allows these cases to have counterparts for plane analysis of cracks in isotropic materials [2]. To avoid the difficulty in handling a high number of material constants, the compact formulations and procedures of the Lekhnitskii–Eshelby–Stroh formalism [1,3], can be used and this one plays an important role in the consequent analysis of the singular stress field of the notch tip.

In comparison with a crack in homogeneous media, in the case of a joint, the stress singularity exponent is different from 1/2 and can also be generally complex in the case of non-homogeneous notches. The stress near the notch tip is mostly characterized by more singular terms and at the same time each singular term covers a combination of both normal and shear modes of loading [4]. It is a matter of interest to express the magnitudes of all these terms and parameters related to the

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singular stress behavior at the notch tip. The most important of them are the exponents of the stress singularities and corresponding eigenfunctions, which are the solution of the eigenvalue problem developed from the geometry and boundary conditions of the notch tip. The first to be considered was the problem of finding stress singularities of the wedge in the case of the isotropic material [5,6], followed by the problems of cracks along or normal to the interface and to other geometries of isotropic composites, e.g. [7]. What plays an important role in this stress singularity analysis is the complex potential features, especially in the case of anisotropic materials. The possibility of their analytical continuation across the domain boundary was applied to the stress singularity analysis of the interface crack in [8]. The procedure developed in [9] based on the Lekhnitskii–Eshelby–Stroh formalism turned out to be an efficient tool for the singular characterization of non-degenerate anisotropic multi-material corners. A comprehensive commentary about this problem can be found in [1,3].

To overcome the problem of non-convergent values for the generalized stress intensity factors calculated directly from the standard definition of the stress intensity factor, path-independent integrals were proposed, such as the well-known *J*-integral [10]. Regarding the maintenance of the path-independence in the cases of the multi-material stress concentrators, the Ψ-integral introduced in [3,11,12] and based on the reciprocal theorem of Betti and Rayleigh can be applied to the problem of a bi-material notch. Even though its application is conditioned by the knowledge of the so-called auxiliary solution of a particular problem, there is no obstruction to use it in conjunction with the complex potentials, which enable to determine the auxiliary solutions. Besides the particular coefficients of the Williams' asymptotic expansion the Ψ-integral provides the determination of the change of potential energy and consequently the energy release rate of a crack perturbing the notch tip. Following the finite fracture mechanics theory, the finite crack length can be assessed.

The stress singular problem discussed in the following paragraphs is suggested as the plane one, where the stress–strain field near the notch tip is at a small scale yielding. The studied domain is composed of two orthotropic materials perfectly bonded along the positive *x*-axis and traction free along the notch faces. The external boundary could be arbitrarily shaped and obeys arbitrary external loading, even though the presented examples provide the application of the involved theory to geometrically rather simple problems.

2. A two-state Ψ-integral

An effective method which can be used for the generalized stress intensity factor calculation could be based on the method of two-state integrals in combination with the finite element method. The two-state integrals, which are path independent, can be based on the *J*-integral [13,14]. However, the *J*-integral cannot be applied for the calculation of generalized stress intensity factors in the cases of *V*-notches or other generalized stress concentrators, because it is not path independent in these cases. Therefore, it is useful to introduce another two-state integral, so-called Ψ-integral also known as *H*-integral.

The two-state integrals and their special features such as path-independence play an important role in fracture mechanics analysis. The Ψ-integral does not lose its properties even in the study of the crack or notch analysis of multi-materials. The method of the Ψ-integral enables us to determine the local stress field in the vicinity of a crack or notch tip using the deformation and the stress field in the remote points, where the numerical results obtained via any numerical analysis, e.g. finite element one, are more accurate. This efficient method is an implication of the Betti's reciprocity theorem, which states the path independence of the following integral supposing the absence of the body forces and residual stresses,

$$\Psi(\boldsymbol{u},\hat{\boldsymbol{u}}) = \int_{\Gamma} \left[\sigma_{ij}(\boldsymbol{u})n_i\hat{u}_j - \sigma_{ij}(\hat{\boldsymbol{u}})n_iu_j\right]\mathrm{d}s, \quad (i,j=1,2), \tag{1}$$

where the summation convention over indexes i,j is used, Γ is any contour surrounding the notch or crack tip, see Fig. 1, $\boldsymbol{u}$ and $\hat{\boldsymbol{u}}$ are two admissible displacement fields and σ_{ij} are corresponding stress tensor components, $\boldsymbol{n}$ is the outward normal to the domain enclosed by the contour Γ. The Ψ-integral contribution to the singular point analysis is its capability to evaluate

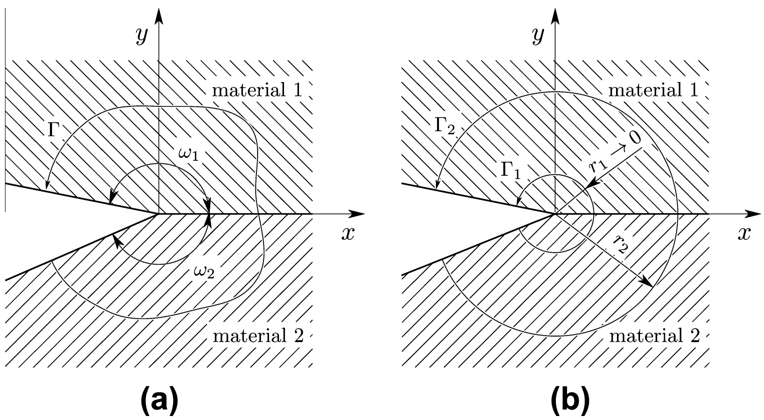

Fig. 1. A geometry and integration paths surrounding the singular point, (a) notation of the geometry, integration path and materials of the bi-material notch, (b) the circular integration paths chosen to the evaluation of the two-state integrals.