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# A multi-scale rate dependent crack model for quasi-brittle heterogeneous materials

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ABSTRACT

A multi-scale numerical approach for modeling cracking in heterogeneous quasi-brittle materials under dynamic loading is presented. In the model, a discontinuous crack model is used at macro-scale to simulate fracture and a gradient-enhanced damage model has been used at meso-scale to simulate diffuse damage. The traction-separation law for the cohesive zone model at macro-scale is obtained from the meso-scale through the discontinuous computational homogenization method. An implicit time integration is used to solve the dynamic problem at the macro-scale while the meso-scale model is solved as a quasi-static problem. The effect of crack opening rate on the macro cohesive law is taken into account by relating the material properties of the meso-scale model to the macro crack opening rate. The objectivity of the model response with respect to the representative volume element size is demonstrated for wave propagation problems. The model is verified by comparison with a direct numerical simulation.



## 1. Introduction

Mixing different materials in order to obtain a better strength to weight ratio and desired material properties is being considered for the design of complex engineering structures for many years. On the other hand, heterogeneities of these materials give rise to difficulties in the design process. For instance, damaging in heterogeneous materials occurs at different spatial and temporal scales and this makes the analysis more complex. Modeling heterogeneous materials using a direct numerical simulation (DNS) in which detailed heterogeneities are modeled directly in the macro-scale can give accurate results but this method needs enormous computational efforts and is in most cases not practical.

Homogenization-based multi-scale methods are widely used to obtain the macroscopic behavior of the heterogeneous materials by averaging local-scale properties. Multi-scale methods can be analytical or computational. Analytical methods are limited to simple problems and cannot be applied to more complex structures. Therefore, numerical [27] and computational homogenization [32,9] methods have received significant attention. In computational homogenization methods, heterogeneous material is replaced by a homogeneous substitute with unknown macroscopic constitutive behavior. Then, a representative volume element (RVE) is associated to each material point and the constitutive law is obtained by solving a boundary value problem for the RVE.

Existence of an RVE is a crucial issue in such techniques. A sample volume can be defined as RVE when homogenized properties do not change significantly with varying RVE size. When a standard homogenization method is used, in linear and hardening regimes, the RVE can be defined but in the softening regime an RVE cannot be defined [10]. However, the

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**Nomenclature**

| | |
|---|---|
| $\boldsymbol{B}$ | matrix of derivatives of the shape functions |
| $\boldsymbol{C_M}$ | macroscopic tangent moduli |
| $c$ | gradient damage parameter |
| $c_0, c_1$ | material parameters |
| $\boldsymbol{D_0}$ | homogenized bulk tangent |
| $\boldsymbol{D}$ | elastic moduli tensor |
| $\boldsymbol{f_M^{bulk}}$ | macro-scale model bulk force vector |
| $\boldsymbol{f_M^{coh}}$ | macro-scale model cohesive force vector |
| $\boldsymbol{f_M^{ext}}$ | macro-scale model external force vector |
| $\boldsymbol{f_m^{ext}}$ | meso-scale model external force vector |
| $\boldsymbol{f_m^{int}}$ | meso-scale model internal force vector |
| $f$ | loading function |
| $G_c$ | fracture energy |
| $h$ | height of the RVE |
| $l$ | averaged width of the localization band |
| $l_M$ | macroscopic characteristic length scale |
| $l_c$ | length scale |
| $l_m$ | local-scale length |
| $\boldsymbol{M}$ | mass matrix |
| $\boldsymbol{N}$ | matrix of nodal shape functions |
| $\boldsymbol{n}$ | normal to the crack band |
| $\boldsymbol{T_M}$ | homogenized cohesive tangent |
| $\boldsymbol{t_M}$ | macroscopic cohesive traction |
| $\boldsymbol{u_M}$ | macro-scale model displacement field |
| $\boldsymbol{u_R}$ | total displacement at the right edge of the RVE |
| $\boldsymbol{u_{dam}^0}$ | compatibility displacement |
| $\boldsymbol{u_{dam}}$ | damage opening |
| $v_p$ | wave speed |
| $w$ | width of the RVE |
| $\beta$ | softening slope |
| $\gamma$ | residual stress |
| $\bar{\varepsilon}_{eq}$ | nonlocal equivalent strain |
| $\boldsymbol{\varepsilon_M}, \boldsymbol{\varepsilon_m}$ | macroscopic and local-scale strain tensors |
| $\varepsilon_{eq}$ | local equivalent strain |
| $\kappa_I$ | damage threshold |
| $\kappa_I^0$ | static damage threshold |
| $\lambda$ | macro-scale wave length |
| $\rho_M, \rho_m$ | macro-scale and meso-scale mass densities |
| $\boldsymbol{\sigma_M}, \boldsymbol{\sigma_m}$ | macroscopic and local-scale stress tensors |
| $\omega$ | damage variable |
| $\Omega_d$ | active damage zone volume |
| $\Omega_M$ | macro-scale volume |
| $\Omega_m$ | RVE volume |
| $[\![\boldsymbol{u}]\!]_{\boldsymbol{M}}$ | macro crack displacement jump |

existence of an RVE for quasi-brittle materials with random complex heterogeneous structures is shown by Nguyen et al. [19], in case a failure zone averaging scheme is introduced. After defining the RVE, a statistical analysis can be performed to determine the size of the RVE for overall linear and non-linear properties [10,12,25]. In [20], based on the failure zone averaging scheme, a homogenization model for modeling cohesive cracking in quasi-brittle materials has been developed. Using this scheme, macroscopic cohesive laws, which are independent of RVE size, can be obtained from the local-scale with localized deformation. A continuous–discontinuous scheme which is a combination of the standard computational homogenization scheme and the discontinuous homogenization scheme was also developed [22,23]. A multi-scale cohesive scheme is proposed in [14,16] to simulate failure of structures bonded with heterogeneous adhesives. In this work, Hill's principle of energy equivalence is used to relate homogenized cohesive law of the adhesive layer at the macro-scale to the damage evolution at the micro-scale for which an isotropic rate dependent damage model is used.

Wave propagation problems in heterogeneous materials using the multi-scale method are studied by many researchers. Fish and Chen [6] showed that higher-order homogenization is necessary to account for wave dispersion effects. They solve the problem of secular terms which increase unbounded with time by using a slow temporal scale [4]. They furthermore eliminate the slow time scale dependence by a non-local model in which various order homogenized equations of motion