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Mode III interfacial crack in the presence of couple-stress elastic materials

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ABSTRACT

In this paper we are concerned with the problem of a crack lying at the interface between dissimilar materials with microstructure undergoing antiplane deformations. The micropolar behavior of the materials is described by the theory of couple-stress elasticity developed by Koiter. This constitutive model includes the characteristic lengths in bending and torsion and thus it is able to account for the underlying microstructure of the two materials. We perform an asymptotic analysis to investigate the behavior of the solution near the crack tip. It turns out that the stress singularity at the crack tip is strongly influenced by the microstructural parameters and it may or may not show oscillatory behavior depending on the ratio between the characteristic lengths.

1. Introduction

Nowadays, bimaterials are efficiently and widely used in many advanced engineering applications, such as layered composite structures, electronic packaging and thin film coatings. For the prediction of failure of these structures and the assessment of acceptable stress level under the condition experienced during service, it becomes essential to estimate the magnitude and distribution of the interfacial stress and strain fields along the interface and mainly near the tip of interface cracks, which may arise and extend under general loading conditions. In particular, antiplane shear loading condition may frequently occur in the life span of composite structures, both alone or accompanied by plane deformation.

Within the classical LEFM theory, the crack tip fields for an interface crack under antiplane strain are similar to the Mode III crack tip fields in a homogeneous medium [1,2]. In both cases, indeed, the shear stresses on the crack plane are the same in the upper and lower bodies, whereas the out-of-plane displacement is zero on the uncracked region of the crack plane. Thus, it is possible to combine the lower and the upper bodies to obtain equilibrium, without changing displacements or stresses in the two halves. Stresses exhibit Mode III symmetry, but displacements do not and thus crack sliding profiles are not symmetric.

Due to the lack of a length scale, the classical theory of elasticity is not able to characterize the constitutive behavior of brittle materials at the micron scale. This lack is expected to be particularly significant for the analysis of the stress and deformation fields very near the crack tip. For a proper investigations of the crack tip fields at the micron scale it becomes necessary to adopt enhanced constitutive models, which account for the presence of microstructure. A way of doing that consists in the inclusion of one or more characteristic lengths, typically of the same order of the compositional grain size, generally few microns, for many advanced materials. The indeterminate theory of couple-stress elasticity (CSE) developed by Koiter [3] involves the material characteristic lengths in bending and torsion. It is sufficiently accurate to simulate the

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Nomenclature

$G_{\pm}$	shear modulus (+/− stands for upper/lower half-plane)
$\nu_{\pm}$	Poisson's ratio
$l_b^{\pm}$	material characteristic length in bending
$l_t^{\pm}$	material characteristic length in torsion
$l_{\pm}$	material characteristic length (Koiter's notation)
$\eta_{\pm}$	ratio between the characteristic lengths in bending and torsion (Koiter's notation, $-1 < \eta_{\pm} \leqslant 1$)
$\boldsymbol{u}$	displacement field
w	out-of-plane displacement
$\boldsymbol{\varphi}$	rotation vector
$\boldsymbol{\epsilon}$	strain tensor
$\boldsymbol{\chi}$	curvature tensor
$\boldsymbol{t}$	nonsymmetric stress tensor
$\boldsymbol{\sigma}$	symmetric part of the stress tensor
$\boldsymbol{\tau}$	skew-symmetric part of the stress tensor
$\boldsymbol{\mu}$	couple-stress tensor
$\boldsymbol{p}$	reduced tractions vector
$\boldsymbol{q}$	reduced couple-stress tractions vector
λ	order of stress singularity

behavior of materials at the micron scale as well as the size effects occurring at distances to the crack tip comparable to characteristic lengths, but it is also simple enough to allow the achievement of closed-form solutions.

Although the presence of the microstructure is expected to modify the interface crack tip field with respect to the classical solution of the LEFM, no analytical investigations have been so far performed about the problem of an antiplane crack along the interface between micropolar and classical elastic materials (the only related work regards a crack terminating perpendicular to the interface [4]). Most of related studies available in literature instead concern the problem of an interface crack under plane deformations, e.g., Itou [5] examined the effect of couple-stresses on the strain-energy release rate for an interface crack loaded by an internal pressure, neglecting somehow the oscillatory behavior of the crack tip fields.

In order to provide an experimental basis for studying the interfacial behavior of a bimaterial specimen under shear loading, Kang et al. [6] applied a method which combines moiré interferometry with phase shift and image processing to measuring the interfacial displacement and strain fields within the interfacial region. Their experimental results show that there is a boundary layer characteristic with a peak value of shear strain and high gradient of rotation angle in the interfacial region. Their study also shows that similar results can be analytically predicted by means of the couple-stress theory, considering the additional freedom of the rotation angle effect.

Hutapea et al. [7] investigated the micro-stress generated along a fiber/matrix interface under generalized plane deformation, which are expected to dominate the failure initiation process in composite laminate. They showed that the micropolar theory is able to capture the interface micro-stress accurately.

A small number of interface crack problems have been investigated by using the strain gradient theory of plasticity [8,9]. In particular, Hao and Liu [8] analyzed the crack propagation in bimaterial systems showing that high stress triaxiality always occurs on the softer material, which may promote ductile damage and facilitate crack growth. Chen and Wang [9] explored the interface crack tip fields at micron scales under plane strain conditions. Their numerical investigations show that the singularity of stresses in the strain gradient theory slightly exceeds or equals to the square-root singularity independently of the material hardening exponents. Askes and Gitman [10] showed numerically that in gradient elasticity no singular behavior is found for a crack terminating perpendicular to the interface.

The analysis of singular stress concentration in homogeneous micropolar elastic solids shows that several pathological predictions of classical elasticity in singular stress concentration problems are altered, mitigated, or possibly eliminated when couple-stresses are taken into account [11]. In particular, the problem of a Mode III crack in a homogeneous materials modeled by the couple-stress elastic theory was first analyzed by Zhang et al. [12] and later by Geogiadis [13] by considering a single characteristic length. The results obtained therein indicate that the skew-symmetric stress components have $r^{-3/2}$ singularity near the crack tip, where r is the distance to the crack tip. Although this singularity is much stronger than the conventional square-root singularity, it does not violate the boundness of strain-energy surrounding the crack tip and leads to a finite energy release rate. Their asymptotic analysis also provides a negative out-of-plane displacement ahead of the crack tip. This unphysical result is due to the exclusion of the lowest order terms for the displacement and symmetric stress components, which do not contribute to the energy release rate.

The effects of both characteristic lengths in bending and torsion and a complete investigation of the crack tip fields under Mode III loading condition in homogeneous CSE materials have been properly addressed in a recent work by Radi [14]. The roles of both characteristic lengths are therein examined in detail and their influence on the crack tip is analytically explored by using Fourier transform and the Wiener–Hopf method. The asymptotic and full-field analyses show that the symmetric stress is finite at the crack tip, whereas the skew-symmetric stress is negative and strongly singular. Ahead of the crack tip