



# Buckling enhancement of laminated composite structures partially covered by piezoelectric actuators

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## ABSTRACT

The buckling (eigenvalue) problem of biaxially compressed laminated plates and shallow cylindrical panels having two symmetric piezoelectrical (PZT) patches on the top and the bottom of laminates is considered. The analysis is carried out with the use of the classical laminate theory and of the first order shear deformation theory. The variable thickness of structures (the local positions of PZT patches) is described by piecewise constant step functions in one direction or in both directions due. Three different methods of the solution of the linear eigenvalue problem are proposed: the exact analytical solution, the approximate solution based on the definition of the Rayleigh quotient and the numerical 3D finite element analysis. For the approximate Donnell's theory of shallow panels two variational formulations of the eigenvalue problem are derived in the form of the Hu-Washizu functional (the Airy stress functions and transverse normal displacements) and in the form of the Legendre functional (displacements). The influence of geometric parameters of composite panels and PZT patches, piezoelectric effect, external electric voltage and laminate configurations (symmetric angle-ply and cross-ply laminates) on buckling characteristics are discussed in detail. The analysis demonstrates evidently that the use of the local piezopatches should be considered as the buckling problem for structures with the non-uniform thickness distribution. The appropriate use of the local PZT patches should be always combined with the appropriate choice of the best (optimal) laminate configuration. The formulation system developed is suitable to other shell theories and to account for the analysis of thermal effects or the imperfection sensitivity.

## 1. Introduction

With the rapid development of piezoelectric materials and their versatile applications in engineering systems, studies of piezoelectric systems have drawn much attention in recent years. Developments in adaptive composite structures incorporating integrated piezoelectric elements open the possibility to adaptively modify the structural behaviour offering potential benefits in a wide range of engineering applications such as vibration suppression, shape control, precision positioning and buckling control, among others. Smart materials are usually attached or embedded into structural systems to enable these structures to sense disturbances, process the information and evoke reaction at the actuators, possibly to negate the effect of the original disturbances. Thus, smart materials respond to environmental stimuli and for that reason they are called responsive materials (response to an applied mechanical stress – *direct piezoelectric effect* or to an external voltage – *converse piezoelectric effect*). The general requirements expected in these materials that integrate the functions sensing, actuation, logic and control include full integration of all functions in the system and

intelligent operational system.

The piezoelectric sensors/actuators (S/As) have to be of suitable size and placement to ensure maximum effectiveness and efficiency. The problem of finding the optimal size and location of S/As is very challenging. The issues of S/A location and geometry, and their optimal selections with respect to certain performance criteria, have drawn much attention due to their importance in structural sensing and control (Muc and Stawiarski, 2012; Stawiarski et al., 2016). Tauchert et al. (2000) presented a review of theoretical developments in the piezoelectricity relevant to adaptive composite structures, addressing also the structural control via piezoelectric actuation. Reddy et al. (Reddy, 1984, 2004; Reddy and Liu, 1985) proposed a higher-order shear deformation theory, in which a parabolic distribution of transverse shear strains through the shell thickness was followed. The related formulation has been frequently applied in the static, buckling and vibration analysis for composites and piezoelectric structures (Arciniega et al., 2004; Arciniega and Reddy, 2005; Shen, 2009; Bagherizadeh et al., 2011; Mirzavand et al., 2013; Sun and Tong, 2001; Muc, 2018a).

Up to now, a lot of research effort has been concentrated on beam,

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plate or cylindrical shell structures for optimal vibration control (Han and Lee, 1999; Muc and Kędziora, 2010, 2012; Kędziora, 2013; Jin et al., 2005; Onoda and Hanawa, 1993; Agrawal and Treanor, 1999; Chee et al., 2002a, 2002b; Muc, 2018b; Kruzelecki and Stawiarski, 2010). However, the investigation of the optimal buckling design of composite adaptive plates with piezoelectric actuators of structures has seldom been carried out. Mota Soares et al. (Franco Correia et al., 2003) maximized buckling loads for composite plates with locally distributed piezoelectric actuators. Some previous works on the enhancement of the buckling behaviour of composite structures by using piezoelectric elements include those of Chandrashekhara and Bhatia (1993), Faria and Almeida (Faria, 2004), Almeida et al. (2012), among others. Batra and Geng (2001) studied the transient elastic deformations of a plate with piezoceramic elements bonded to the top and bottom surfaces and analysed the effect of the shape and size of the piezoceramic actuators on increasing the buckling load of the plate. Loughlan et al. (2002), Abramovich (2011) carried out experimental tests which illustrate the feasibility of buckling control in composite structural elements using induced strain actuation by using shape memory actuators. Shen (2002) presented a post-buckling analysis for cross-ply laminated cylindrical shells with piezoelectric actuators subjected to the combined action of external pressure and heating and under different electric voltage situations. Kundu et al. (2007) analysed post-buckling behaviour of bimorph cylindrical, conoidal and spherical laminated shells. The results obtained in the above-mentioned papers demonstrated the stiffening effect of piezoelectric patches on the elastic stability of laminated structures.

It should be emphasized that up to now the majority of works dealing with buckling analysis of laminated structures with PZT layers presents the results for so-called bimorph structures. In this study, it is assumed that actuators cover parts of the laminated structures. Since the thicknesses of the manufactured PZT plates can be in a range from 0.2 mm up to 40 mm (comparable to the laminate thicknesses) the buckling loads should to be evaluated for variable (non-uniform) thickness distribution. A specified and determined voltage is applied to the actuators as an input – Fig. 1.

The basic structure to be investigated is shown in Fig. 1 where a panel, either flat or cylindrical, has two equal piezoelectric layers attached to the top and bottom surfaces. This voltage makes the laminate structure deform. The objective is the maximization of the buckling load. The design variables are the discrete locations of the piezoelectric actuators and the number of plies oriented at  $0^\circ$  and  $90^\circ$  also the in the composite layers (cross-ply laminates) or the ply orientation in the case of symmetric angle-ply orientations. The optimal distributions of the piezoelectric patches based on buckling modes are investigated. As an illustration, numerical examples of a simply supported plate and a simply supported cylindrical panel are presented to demonstrate the feasibility of this method. A special attention is focused on the formulation of the governing relations in order to discuss their influence on the buckling results.

## 2. Modeling and formulation

### 2.1. Physical relations

Exerting mechanical stress on piezoelectric materials results in electrical field creation and conversely exerting an electrical field results in mechanical strain in piezoelectric materials. Stress  $\sigma$ , strain  $\epsilon$  from mechanical points of view are coupled with electrical displacements  $D$  and electrical field  $E$ . Modeling of composite structures having smart piezoelectric sensors or actuators is very similar to that for conventional composite layered structures, however, there is one difference reflected in the constitutive laws in the form of the electromechanical coupling. It affects also the additional complexities in the FE formulation. The constitutive model for laminated panels with embedded piezoelectric sensors/actuators (S/A(s)) is established in the global rectangular coordinate  $x$ - $y$ - $z$  system. In the matrix form it is given by:

$$\begin{bmatrix} \sigma \\ D \end{bmatrix} = [\bar{C}] \begin{bmatrix} \epsilon \\ E \end{bmatrix} = \begin{bmatrix} [C] & [-e] \\ [e]^T & [\mu] \end{bmatrix} \begin{bmatrix} \epsilon \\ E \end{bmatrix} \quad (1)$$

$$\begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_4 \\ \sigma_5 \\ \sigma_6 \\ D_1 \\ D_2 \\ D_3 \end{Bmatrix} = \begin{bmatrix} \bar{Q}_{11} & \bar{Q}_{12} & 0 & 0 & \bar{Q}_{16} & 0 & 0 & \bar{e}_{31} \\ \bar{Q}_{12} & \bar{Q}_{22} & 0 & 0 & \bar{Q}_{26} & 0 & 0 & \bar{e}_{32} \\ 0 & 0 & \bar{Q}_{44} & \bar{Q}_{45} & 0 & \bar{e}_{14} & \bar{e}_{24} & 0 \\ 0 & 0 & \bar{Q}_{45} & \bar{Q}_{55} & 0 & \bar{e}_{15} & \bar{e}_{25} & 0 \\ \bar{Q}_{16} & \bar{Q}_{26} & 0 & 0 & \bar{Q}_{66} & 0 & 0 & \bar{e}_{36} \\ 0 & 0 & \bar{e}_{14} & \bar{e}_{15} & 0 & \mu_{11} & 0 & 0 \\ 0 & 0 & \bar{e}_{24} & \bar{e}_{25} & 0 & 0 & \mu_{22} & 0 \\ \bar{e}_{31} & \bar{e}_{32} & 0 & 0 & \bar{e}_{36} & 0 & 0 & \mu_{33} \end{bmatrix} \begin{Bmatrix} \epsilon_1 \\ \epsilon_2 \\ \epsilon_4 \\ \epsilon_5 \\ \epsilon_6 \\ E_1 \\ E_2 \\ E_3 \end{Bmatrix} \quad (2)$$

The symbols having the bar over them have the standard mechanical interpretation, i.e. stresses  $[\sigma]$  and strains  $[\epsilon]$ . The coefficients in  $[Q]$  matrix can be calculated from the elastic moduli, Poisson's ratios, piezoelectric moduli and dielectric constants of the lamina. Required equations are given in the Appendix for readers' convenience.  $[D]$  is the vector of electric displacements (three components),  $[e]$  is the matrix of piezoelectric coefficients of size  $6 \times 3$ ,  $[\mu]$  is the permittivity matrix of size  $3 \times 3$  and  $[E]$  is the applied electric field in three coordinate directions. The electric field is defined as the gradient of the electric potential  $\Phi_{el}$ , i.e:

$$[E] = -\text{grad } \Phi_{el}$$

$$\begin{Bmatrix} E_x \\ E_y \\ E_z \end{Bmatrix} = \begin{Bmatrix} -\frac{\partial \Phi_{el}}{\partial x} \\ -\frac{\partial \Phi_{el}}{\partial y} \\ -\frac{\partial \Phi_{el}}{\partial z} \end{Bmatrix} \quad (2)$$

The boundary condition on the electric field can be written as  $E_x = E_y = 0$  on the top and bottom faces of the panel. Further, a linear function for electrostatic potential through actuating layer in the  $z$

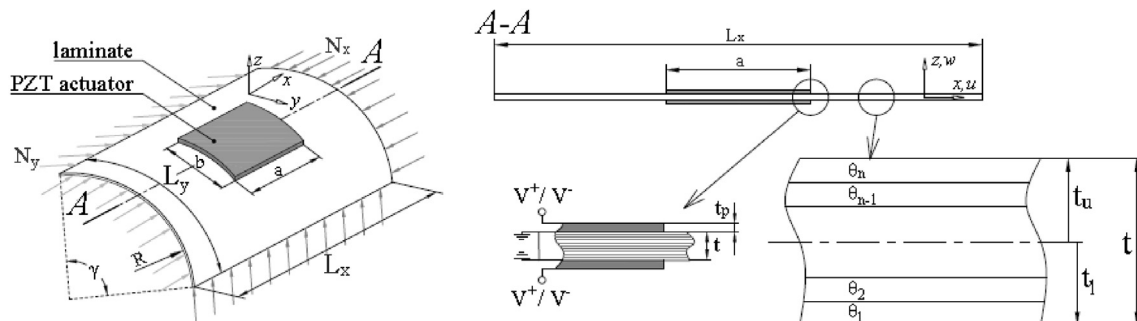


Fig. 1. Illustration of: a) the laminated panel and b) the laminate cross-section.

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