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Microscopic instabilities and elastic wave propagation in finitely deformed laminates with compressible hyperelastic phases

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ABSTRACT

In this paper, we study the elastic instability and wave propagation in compressible layered composites undergoing large deformations. We specifically focus on the role of compressibility on the onset of instability, and elastic wave band gaps (forbidden frequency ranges) in finitely deformed buckled laminates. We employ the Bloch-Floquet analysis to study the influence of compressibility on the onset of instability and the corresponding critical wavelengths. Then, the obtained information about the critical wavelengths is used in the subsequent numerical postbuckling simulations. By application of the Bloch wave numerical analysis implemented in the finite element code, we investigate the elastic wave band gaps of buckled layered composites with compressible phases.

The compressible laminates require larger strains to trigger mechanical instabilities. This results in lower amplitudes of instability induced wavy patterns in compressible laminates as compared to incompressible layered materials. The instability induced wavy patterns give rise to tunability of the widths and locations of shear wave band gaps (that are not tunable by deformation in LCs with neo-Hookean phases in the stable regime); this tunability, however, is not significant in comparison to the tunability of the pressure wave band gaps. Thus, the complete band gaps (frequency ranges where neither shear nor pressure wave can propagate) can be controlled by deformation in both stable and post-buckling regimes.

1. Introduction

Design of microstructured metamaterials for manipulating elastic wave propagation has drawn considerable attention (Babaee et al., 2016; Bigoni et al., 2013; Celli et al., 2017; Celli and Gonella, 2015; Chen and Elbanna, 2016; Chen and Wang, 2016; Harne and Urbanek, 2017; Matlack et al., 2016; Miniaci et al., 2016; Srivastava, 2016; Trainiti et al., 2016; Xu et al., 2015; Zhu et al., 2014; Zigoneanu et al., 2014). These new materials can potentially serve for enabling various applications, such as wave guide (Casadei et al., 2012), vibration damper (Javid et al., 2016), cloaking (Zhang et al., 2011), and subwavelength imaging (Wood et al., 2006; Zhu et al., 2011). Recently, soft metamaterials with reconfigurable microstructures in response to external stimuli, such as mechanical load (dell'Isola et al., 2016; Galich et al., 2017a; Li et al., 2016; Meaud and Che, 2017; Zhang and Parnell, 2017), electric and/or magnetic field (Bayat and Gordaninejad, 2015; Galich and Rudykh, 2017, 2016; Gei et al., 2011; Huang et al., 2014; Jandron and Henann, 2017; Yang and Chen, 2008), attracted significant interest for tuning elastic wave propagation. Moreover, the elastic instability induced buckling phenomena, giving rise to a sudden change in microstructure, have been demonstrated to be greatly instrumental for the design of switchable phononic crystals. Thus, Bertoldi and Boyce (2008a, 2008b) introduced the concept of instability assisted elastic wave band gaps (BGs) control in soft elastomeric materials with peridically distributed circular voids (Shan et al., 2014; Wang et al., 2014, 2013). Rudykh and Boyce (2014) showed that the elastic instability induced wrinkling of interfacial layers could be utilized to control the BGs in deformable layered composites (LCs). In this work, we analyze the phenomena with specific focus on the influence of the constituent compressibility on the instabilities and elastic wave BGs of finitely deformed neo-Hookean laminates in the postbuckling regime.

The important work on the stability of layered and fiber composites by Rosen (1965), considered stiff layers embedded in a soft matrix as elastic beams on an elastic foundation, and derived an explicit expression to predict the critical buckling strain. Parnes and Chiskis (2002) revisited the instability analysis in linear elastic LCs, and they found that the buckling strain of dilute composites that experienced microscopic instability was constant, while for the macroscopic case,

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the buckling strain agreed with the results of Rosen (1965). Triantafyllidis and Maker (1985) analyzed the onset of instability in finitely deformed periodic layered composites. They demonstrated the existence of the microscopic and macroscopic (or long wave) instabilities by employing the Bloch-Floquet analysis (Geymonat et al., 1993), along with the loss of ellipticity analysis that is typically used to detect the onset of macroscopic instability (Merodio and Ogden, 2005, 2003, 2002). Nestorovic and Triantafyllidis (2004) investigated the interplay between macroscopic and microscopic instability of hyperelastic layered media subjected to combinations of shear and compression deformation. Micromechanics based homogenization was utilized to predict the macroscopic instability of transversely isotropic fiber composites with hyperelastic phases (Agoras et al., 2009; Rudykh and Debotton, 2012). Recently, Gao and Li (2017) showed that the wavy patterns of the interfacial layer could be tuned by the interphase between the interfacial layer and soft matrix. Slesarenko and Rudykh (2017) implemented the Bloch-Floquet technique into the finite element based code and examined the macroscopic and microscopic instability of periodic hyperelastic 3D fiber composites. More recently, Galich et al. (2018) focused on the influence of the periodic fiber distribution on instabilities and shear wave propagation in the hyperelastic 3D fiber composites. Furthermore, the microscopic and macroscopic instability phenomena of multi-layered composites under plane strain conditions were observed in experiments via 3D-printed layered materials (Li et al., 2013). Slesarenko and Rudykh (2016) experimentally showed that the wavy patterns in LCs with visco-hyperelastic constitutes could be tuned by the applied strain rate. Li et al. (2018a) experimentally realized the instability development in periodic 3D fiber composites. Through these studies, the role of stiff fiber reinforcement on the stability of composites has been well understood; in particular, the composites with stronger reinforcement (with higher shear modulus contrasts or with larger fiber volume fractions) are more prone to instabilities. However, the role of phase compressibility on the instability development and post-buckling behavior of hyperelastic laminates has not been examined.

In the first part of our paper, we will focus on the influence of phase compressibility on the onset of instability and critical wavelengths that define the postbuckling patterns of the microstructure. We note that it is possible to use the estimates for the onset of instability and critical wavelengths based on the linear elasticity theory (Li et al., 2013; Rudykh and Boyce, 2014); this, however, does not fully account for the nonlinear effects of finite deformations. To take into account these effects, we perform the instability analysis superimposed on finite deformations. The obtained information about the critical wavelengths is further used in the analysis presented in the second part of the paper, where the elastic waves in the postbuckling regime are analyzed.

Rytov (1956) derived explicit dispersion relations for elastic waves propagating perpendicular to the layers showing the existence of the elastic wave BGs (or stop bands) in LC frequency spectrum. Wu et al. (2009), and Fomenko et al. (2014) investigated the elastic wave BGs of layered media with functionally graded materials. Recently, Srivastava (2016) predicted the appearance of negative refraction at the interface between layered composite media and homogeneous material. More recently, Slesarenko et al. (2018) showed that negative group velocity can be induced by deformation in hyperelastic composites in the stable regime near elastic instabilities. Galich et al. (2017a) obtained explicit expressions for shear and pressure long waves in finitely deformed LCs with isotropic hyperelastic phases. Moreover, based on the analysis by Rytov (1956), Galich et al. (2017a) extended the classical results to the class of finitely deformed hyperelastic laminates. In particular, Galich et al. (2017a) show that the shear wave BGs are independent of the applied deformation in neo-Hookean laminates. In addition, the results of Galich et al. (2017a) demonstrate that the pressure wave BGs can be tuned by deformation, mostly via the change in the thickness of the layers. In this work, we examine the elastic wave propagation in finitely deformed neo-Hookean laminates in the *postbuckling* regime, and we specifically focus on the influence of material compressibility.

The paper is structured as follows: Section 2 presents the theoretical background for finite elastic deformation and small amplitude motions superimposed on the finitely deformed state. The numerical simulations, including the procedures to detect the onset of instability and perform postbuckling analysis, are described in Section 3. The results are presented in Section 4, which is divided into two subsections. Section 4.1 is devoted to the analysis of the influence of the constituent compressibility on the onset of instability; and Section 4.2 presents the analysis of elastic wave propagation in finitely deformed compressible LCs in the postbuckling regime. Section 5 concludes the study with a summary and discussion.

2. Theoretical background

Consider a continuum body and identify each point in the undeformed configuration with its position vector $\mathbf{X}$. When the body is deformed, the new location of the corresponding point is defined by mapping function $\mathbf{x} = \boldsymbol{\chi}(\mathbf{X}, t)$. The deformation gradient is defined by $\mathbf{F} = \partial\mathbf{x}/\partial\mathbf{X}$ and its determinant is $J = \det(\mathbf{F}) > 0$. For hyperelastic materials whose constitutive behaviors are described in terms of strain energy density function $W(\mathbf{F})$, the first Piola-Kirchhoff stress tensor is given by

$$\mathbf{P} = \frac{\partial W(\mathbf{F})}{\partial \mathbf{F}}. \tag{1}$$

In the absence of body forces, the equations of motion can be written in the undeformed configuration as

$$\text{Div}\,\mathbf{P} = \rho_0 \frac{D^2\boldsymbol{\chi}}{Dt^2}, \tag{2}$$

where Div(•) represents the divergence operator in the undeformed configuration, $D(\bullet)/Dt$ is the material time derivative, and ρ_0 denotes the initial material density. When deformation is applied quasi-statically, Eq. (2) reads

$$\text{Div}\,\mathbf{P} = 0. \tag{3}$$

Next we consider small amplitude motions superimposed on an equilibrium state (Destrade and Ogden, 2011; Ogden, 1997). The equations of the incremental motion are

$$\text{Div}\,\dot{\mathbf{P}} = \rho_0 \frac{D^2\mathbf{u}}{Dt^2}, \tag{4}$$

where $\dot{\mathbf{P}}$ is an incremental change in the first Piola-Kirchhoff stress tensor and $\mathbf{u}$ is an incremental displacement. The incremental change in deformation gradient is given by

$$\dot{\mathbf{F}} = \text{Grad}\,\mathbf{u}, \tag{5}$$

where Grad (•) represents the gradient operator in the undeformed configuration.

The linearized constitutive law can be expressed as

$$\dot{P}_{ij} = \mathscr{A}_{0ijkl}\dot{F}_{kl}, \tag{6}$$

where $\mathscr{A}_{0ijkl} = \partial^2 W/\partial F_{ij}\partial F_{kl}$ is the tensor of elastic modulus. Substitution of Eqs. (5) and (6) into Eq. (4) yields

$$\mathscr{A}_{0ijkl}\frac{\partial^2 u_k}{\partial X_j \partial X_l} = \rho_0 \frac{D^2 u_i}{Dt^2}. \tag{7}$$

In the updated Lagrangian formulation, Eq. (7) reads

$$\mathscr{A}_{ijkl}\frac{\partial^2 u_k}{\partial x_j \partial x_l} = \rho \frac{\partial^2 u_i}{\partial t^2}, \tag{8}$$

where $\mathscr{A}_{ipkq} = J^{-1}\mathscr{A}_{0ijkl}F_{pj}F_{ql}$ and $\rho = J^{-1}\rho_0$.