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Analysis on two types of internal resonance of a suspended bridge structure with inclined main cables based on its sectional model

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ABSTRACT

This study investigated the internal resonance phenomenon of a suspended bridge structure with a 6-Degree-of-Freedom sectional model. The primary resonance of the second mode of the system under harmonic excitation is firstly studied. The one-to-two internal resonance between the second and third modes, and one-to-three internal resonance between the second and fourth modes may be induced if the corresponding natural frequency ratios are close to 2.0 and 3.0, respectively. Numerical analysis also shows that the two-to-one internal resonance between the third and second modes can be induced under different scenarios of excitation. The first one happens with the second mode being excited, resulting in two response peaks in the frequency response curves. The second one may occur when the third mode is excited with sufficient large excitation where most of the energy input will be transmitted to the second mode. Hopf bifurcation can also be found in the frequency response curve of the system. Lastly, the three-to-one internal resonance between the fourth and second modes is also found when the second mode is excited. The response of the second mode is slightly reduced with a distinct increase in the response of the fourth mode due to the internal resonance. All these behaviors of this dynamic system indicate the meaningful role played by a variety of internal resonances in the design of mega-scale cable-supported bridge structure.

1. Introduction

The dynamics of long span cable-supported bridges is always a major topic of research in civil engineering. Continuum model and sectional model are two modeling approaches for the study. Sectional models have been demonstrated to be capable to illustrate the interaction between oscillations in the torsional and vertical directions, though in an approximate and analytical form.

The most popular sectional model is a two-Degrees-of-Freedom (2-DoFs) system, which can demonstrate the vibration of one vertical mode and one rotational mode of the bridge deck (Jacover and McKenna, 1994; Doole and Hogan, 2000). Gao and Zhu (2015) studied the nonlinear mechanical stiffness and damping of this sectional model. De Freitas et al. (2014a; 2014b) used the Lazer-McKenna model (Lazer and McKenna, 1990) to study the nonlinear properties of the system under periodic external forces. The 2-DoFs model is relatively simple and it could not give a more detailed description of the bridge behavior.

Other researchers extended the 2-DoFs model into a 4-DoFs model by considering additional vertical motion of the main cables on two sides. Plaut and Davis (2007) adopted the 4-DoFs model in their investigation on the vertical and rotational motions of the bridge deck together with the vertical motion of the cables when the stiffness symmetry of the system about the centerline of deck is suddenly lost. More recently, a multi-body model was proposed to model the bridge section of long span bridges for the linear and nonlinear dynamic studies (Lepidi and Gattulli, 2014, 2016; Lepidi and Piccardo, 2014). In these studies, the bridge section was modeled by a 4-DoFs system accounting for both the vertical and torsional motions of the bridge deck as well as the transversal motion of a pair of hangers or stay cables. Although the vertical motions of hangers were included in the governing equations of the system, they are neglected in the subsequent analysis due to the fact that the elastic longitudinal stiffness of the hangers is much higher than the geometric lateral stiffness. This model has been adopted in the investigation of interaction between the motions of the bridge deck and stay cables, and the internal resonance between the local modes and global modes. The study of dynamic behavior of the cable-supported bridges has been developed into a new area of research since.

The 2-DoFs and 4-DoFs models have been used to study the dynamic interaction in existing types of cable-supported bridges. New types of

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Nomenclature

V_i $(i = 1,2)$	Projection of S-i $(i = 1,2)$ in vertical direction
H_i $(i = 1,2)$	Projection of S-i $(i = 1,2)$ in horizontal direction
D	Half width of the bridge deck
α	Representative inclination angle of cable and hanger $\alpha = \arctan[(H_1 + H_2)/(V_1 + V_2)]$
l_i $(i = 1,2,3)$	Length of spring S-i $(i = 1,2,3)$
m	Mass of cable
M	Mass of bridge deck
J	Moment of Inertia of bridge deck
k_i $(i = 1,2,3,4)$	Stiffness of spring S-i $(i = 1,2,3,4)$
u_i $(i = 1,2)$	Horizontal DoF of m
v_i $(i = 1,2)$	Vertical DoF of m
y	Vertical DoF of M
θ	Rotational DoF of M
F_{cs}	Internal force of S-1 under static condition
F_{hs}	Internal force of S-2 under static condition
φ_{ci} $(i = 1,2)$	Angle between S-1 and the vertical plane
φ_{hi} $(i = 1,2)$	Angle between S-2 and the vertical plane

suspension bridge have been developed in the last two decades, such as the one shown in Fig. 1(a). The cables spatially support the deck in both the vertical and lateral directions. Since the main cables are inclined to the horizon, its motion is not confined to the vertical or horizontal direction only. A 6-DoFs sectional model has been proposed by the authors to model the structure including dynamic behavior of the main cables (Hui et al., 2018). The effect of the cable inclination on the nonlinear dynamic properties of the system in the primary resonance of the first two modes was investigated.

Many studies have shown that the internal resonance is a big concern in analyzing the dynamic responses of a multiple DoFs system (Kang et al., 2017; Gattulli and Lepidi, 2003). The dynamics properties of a suspension bridge with inclined main cables are further studied in this paper in the investigation of the internal resonance between different vibration modes. This paper is organized as follows. The 6-DoFs model is briefly firstly introduced, and the modal property of the system is examined with study on the effect of system parameters on the modal frequency ratios between different modes. The primary resonance of the second mode (torsional mode of deck) indicates the possibility of internal resonance between the second mode and other higher modes. The two-to-one internal resonance between the third and second modes is further investigated with discussions on the phenomena under different scenarios of excitation. Three-to-one internal resonance between the fourth and second modes then follows.

2. The sectional model

The bridge sectional model (Hui et al., 2018) comprises of three masses linked by six springs as shown in Fig. 1(b). The masses are from the sectional bridge deck and the two cables. The six springs are grouped into four types: S-1 denotes the in-plane stiffness of the main cable; S-2 denotes the axial stiffness of the hanger; S-3 and S-4 denote respectively the vertical and torsional stiffness provided by the bridge deck. This model is adopted in the following studies on motion in the y-z plane where the contributions from the cables, hangers and deck to the dynamic interaction of vibration modes of the system can be accounted for. The variables in the figure are defined as follows:

The equations of motion governing the free undamped vibration of the system can be written as

$$M\ddot{y} + k_3 y - (F_{hs} + F_{hd1})\cos(\phi_{h1}) - (F_{hs} + F_{hd2})\cos(\phi_{h2}) + Mg = 0 \tag{1a}$$

$$\begin{aligned} J\ddot{\theta} + k_4\theta - (F_{hs} + F_{hd1})\cos(\phi_{h1})D\cos\theta + (F_{hs} + F_{hd2})\cos(\phi_{h2})D\cos\theta \\ + (F_{hs} + F_{hd1})\sin(\phi_{h1})D\sin\theta + (F_{hs} + F_{hd2})\cos(\phi_{h2})D\sin\theta = 0 \end{aligned} \tag{1b}$$

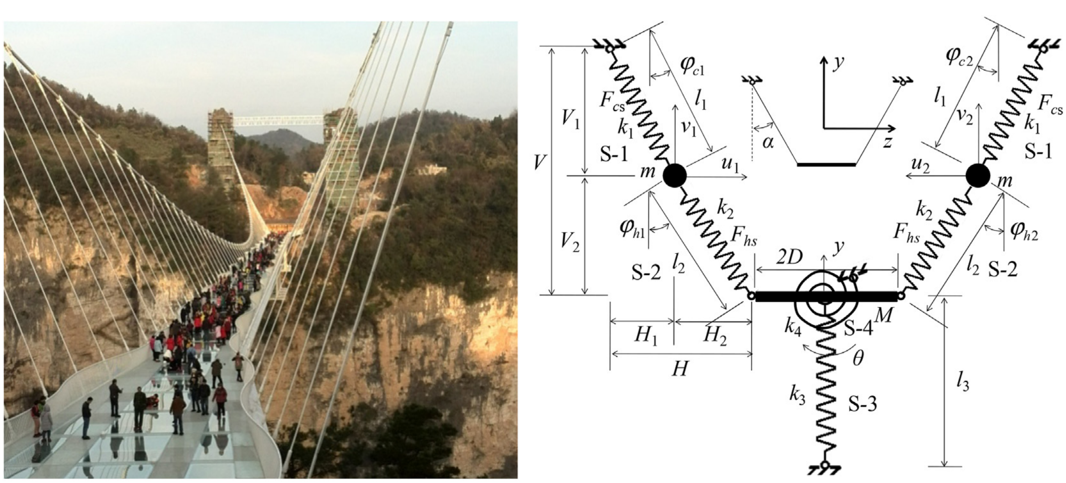
$$m\ddot{u}_1 + (F_{cs} + F_{cd1})\sin(\phi_{c1}) - (F_{hs} + F_{hd1})\sin(\phi_{h1}) = 0 \tag{1c}$$

$$m\ddot{v}_1 + mg - (F_{cs} + F_{cd1})\cos(\phi_{c1}) + (F_{hs} + F_{hd1})\cos(\phi_{h1}) = 0 \tag{1d}$$

$$m\ddot{u}_2 + (F_{cs} + F_{cd2})\sin(\phi_{c2}) - (F_{ls2} + F_{ld2})\sin(\phi_{h2}) = 0 \tag{1e}$$

$$m\ddot{v}_2 + mg - (F_{cs} + F_{cd2})\cos(\phi_{c1}) + (F_{hs} + F_{hd2})\cos(\phi_{h1}) = 0 \tag{1f}$$

where F_{cdi} and F_{hdi} $(i = 1,2)$ are respectively the dynamic forces in springs S-1 and S-2. When the trigonometric functions are expanded into Taylor series up to the second order, these equations of motion can be rewritten in the following forms as



(a) Cable-supported bridge in Zhangjiajie, China (b) Sectional model

Fig. 1. Cable-supported bridge with spatially inclined cables.