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Evaluation of intensities of singularity at three-dimensional piezoelectric bonded joints using a conservative integral

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ABSTRACT

A conservative integral based on the Betti reciprocal principle is extended for calculating intensities of singularity at vertices of interfaces in three-dimensional piezoelectric bonded joints. Regarding this method, eigenanalysis, formulated using a three-dimensional finite element method (FEM), is used for calculating the orders of stress singularity, angular functions of mechanical displacements, stresses, electric displacements and electric potentials. This method was initially developed for bonded joints between isotropic materials. For piezoelectric bonded joints, in addition to the stress singularities, the singularities associated with the electric field are also important. Hence, both the singularities associated with the mechanical stresses and the electric displacements are investigated and discussed in this study. The interface between piezoelectric and isotropic materials with one-term of stress singularity, and the interface between piezoelectric and piezoelectric materials with three-terms of stress singularity, are analyzed. Several models with various element sizes and integral areas are examined to investigate the influence of mesh refinement and integral area on the accuracy of the results. In order to summarize the results of one-term and three-terms of stress singularity in a general form, a unified singular equation is proposed.

1. Introduction

Dissimilar material bonded joints have stress singularity at the edge of their interface due to mismatching of material properties across interfaces. A large magnitude of singular stresses at vertices of joints may be a cause of fractures and failures in joints. Hence, it is important to study the singularity behavior of dissimilar material bonded joints. There are two important parameters; (1) the order of stress singularity and (2) the intensity of singularity; those shall both be considered in order to understand the singularity behavior of the bonded joints.

In the present study, we are focusing on the evaluation of the intensity of singularity in three-dimensional piezoelectric bonded joints. Piezoelectric bonded joints have been widely used in numerous engineering and technology products, e.g., sensors or actuators. Recently, advanced numerical methods have been developed to analyze singular stress in piezoelectric materials, e.g., an extended finite element method (Sharma et al., 2013; Liu et al., 2014; Yu et al., 2015), a time-domain boundary element method (Lei and Zhang, 2012), and the M-integral (Motola and Banks-Sills, 2009). There are several studies on the analysis of the orders of singularity of piezoelectric material bonded joints (Chen, 2006; Islam and Koguchi, 2010). However, there are only a few studies on the determination of the intensities of singularity at the piezoelectric bonded joints. For example, Islam and Koguchi (2012) used the boundary element method (BEM) with a curve-fitting technique to determine the intensities of singularities, and Hirai et al. (2012) used the conservative integral to determine the intensity of singularity in a two-dimensional piezoelectric bonded joint.

The conservative integral based on the Betti reciprocal principle is extended to determine the intensities of singularity at a vertex of the interface in piezoelectric bi-material bonded joints. The conservative integral has been proved to be a powerful method for calculating the intensity of singularity. This method was first developed for calculating the stress intensity factors of notched homogeneous bodies (Stern et al., 1976; Sinclair et al., 1984; Carpenter, 1984). After that, Carpenter and Byers (1987), Banks-Sills (1997), and Banks-Sills and Sherer (2002) extended the application of this method to determine the intensity of singularity in two-dimensional dissimilar material joints.

Recently, the conservative integral to determine the intensity of singularity at the three-dimensional vertex of the interface in isotropic bi-material joints was developed by Luangarpa and Koguchi (2014). Following this, the conservative integral was extended for calculating the intensity of singularity along a singular line (Luangarpa and

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Koguchi, 2016). Regarding this method, eigen analysis formulated by a three-dimensional finite element method (FEM) is used to calculate the order of stress singularity, angular functions of displacements and stresses (Pageau and Biggers, 1995; Islam and Koguchi, 2010). To our knowledge, no study on the determination of the intensities of singularity in the three-dimensional piezoelectric bonded joints using the conservative integral has ever been conducted.

For piezoelectric material, the singular field is more complicated than non-piezoelectric material. The singularities associated with the electric field shall be considered. Hence, additional parameters including electric potential and electric displacements are added into the formulation. Two models, which have different material combinations and different geometry, are examined. Model-1 is the bi-material bonded joint composed of piezoelectric material and isotropic material. This model has one-term of singularity. Model-2 is more complicated than Model-1. This model is the bi-material bonded joint composed of piezoelectric material and piezoelectric material with three-terms of singularity. In the cases of multi-terms of singularity, there is difficulty in separating each term of the intensity of singularity from distributions of stresses or electric displacements. Therefore, a major advantage of the conservative integral for this case is that the intensity of singularity for each singularity term can be determined separately. For this model, two cases with different applied loadings; mechanical loading and electrical loading, are investigated.

A unified singular stress equation is proposed in Section 2 for combining cases of one-term and three-terms of stress singularity. The equation consisting of three-terms of stress singularity is used for both models. The singular field in Model-1 is analyzed by adding two more terms, which have no effect on the overall results.

In order to investigate the influence of mesh refinement and integral area on the accuracy of the results, several mesh models with various element sizes and integral areas are examined. Finally, the distributions of stresses and electric displacements obtained using the conservative integral are compared with those obtained using the FEM, in which extremely refined meshes are employed, to confirm the accuracy of the method.

2. Analytical formula

2.1. Stress singularity for piezoelectric bonded

Equations for analyzing elasto-electric problems of piezoelectric materials are presented in this section. The constitutive relations for piezoelectric materials are expressed as follows:

$$\sigma_{ij} = C_{ijkl}\varepsilon_{kl} - e_{kij}E_k, \tag{1}$$

$$D_i = e_{kij}\varepsilon_{kl} + \chi_{ik}E_k, \tag{2}$$

where $(i, j, k, l = 1, 2, 3)$, σ_{ij} is the stress tensor, C_{ijkl} is the elastic constant, ε_{kl} is the strain tensor, e_{kij} is the piezoelectric constant, E_k is the electric field, D_i is the electric displacement vector, and χ_{ik} is the dielectric constant. (Ikeda, 1990)

In the absence of body forces and free charges, the equilibrium equations are given by

$$\sigma_{ij,j} = 0,\ D_{i,i} = 0. \tag{3}$$

The asymptotic stresses around the singular point in the spherical coordinate system can be described as follows:

$$\sigma_{ij}(r, \theta, \phi) = \sum_{n=1}^{m} K_n\left(\frac{r}{L}\right)^{-\lambda_n} f_{ij}^{(n)}(\theta, \phi), \tag{4}$$

where $(i, j = r, \theta, \phi)$, r is the radial distance from the singular point, L is a model length, K_n is the intensity of singularity, λ_n is the order of the stress singularity, f_{ij} is the angular function of stress, and m is the number of singularity term.

The displacement fields are given by

$$u_i(r, \theta, \phi) = \sum_{n=1}^{m} K_n\left(\frac{r^{1-\lambda_n}}{L^{-\lambda_n}}\right) g_i^{(n)}(\theta, \phi). \tag{5}$$

where u_i is the displacement, and g_i is the angular function of displacement.

Equations (4) and (5) are the same as the equations of non-piezoelectric material. For piezoelectric material, electric properties shall be considered. The asymptotic electric displacement and electric potential are written as follows:

$$\sigma_{4j}(r, \theta, \phi) = \sum_{n=1}^{m} K_n\left(\frac{r}{L}\right)^{-\lambda_n} f_{4j}^{(n)}(\theta, \phi), \tag{6}$$

$$u_4(r, \theta, \phi) = \sum_{n=1}^{m} K_n\left(\frac{r^{1-\lambda_n}}{L^{-\lambda_n}}\right) g_4^{(n)}(\theta, \phi), \tag{7}$$

where σ_{4j} and u_4 are the electric displacement and the electric potential, f_{4j} is the angular function of electric displacement, and g_4 is the angular function of electric potential.

In this study, two models with one-term and multi-term of singularities are investigated. The stresses and electric displacements are simplified to be the unified singular equation with three-term of singularities as follows:

$$[\boldsymbol{\sigma}(r, \theta, \phi)] = [\boldsymbol{f}(\theta, \phi)]\left[\left(\frac{r}{L}\right)^{-\lambda}\right][\boldsymbol{K}] \tag{8}$$

where

$$[\boldsymbol{\sigma}(r, \theta, \phi)] = \begin{Bmatrix} \sigma_{rr}(r, \theta, \phi) \\ \sigma_{\theta\theta}(r, \theta, \phi) \\ \sigma_{\phi\phi}(r, \theta, \phi) \\ \tau_{r\theta}(r, \theta, \phi) \\ \tau_{r\phi}(r, \theta, \phi) \\ \tau_{\theta\phi}(r, \theta, \phi) \\ D_r(r, \theta, \phi) \\ D_\theta(r, \theta, \phi) \\ D_\phi(r, \theta, \phi) \end{Bmatrix}, [\boldsymbol{f}(\theta, \phi)] \begin{bmatrix} f_{rr}^{(1)}(\theta, \phi) & f_{rr}^{(2)}(\theta, \phi) & f_{rr}^{(3)}(\theta, \phi) \\ f_{\theta\theta}^{(1)}(\theta, \phi) & f_{\theta\theta}^{(2)}(\theta, \phi) & f_{\theta\theta}^{(3)}(\theta, \phi) \\ f_{\phi\phi}^{(1)}(\theta, \phi) & f_{\phi\phi}^{(2)}(\theta, \phi) & f_{\phi\phi}^{(3)}(\theta, \phi) \\ f_{r\theta}^{(1)}(\theta, \phi) & f_{r\theta}^{(2)}(\theta, \phi) & f_{r\theta}^{(3)}(\theta, \phi) \\ f_{r\phi}^{(1)}(\theta, \phi) & f_{r\phi}^{(2)}(\theta, \phi) & f_{r\phi}^{(3)}(\theta, \phi) \\ f_{\theta\phi}^{(1)}(\theta, \phi) & f_{\theta\phi}^{(2)}(\theta, \phi) & f_{\theta\phi}^{(3)}(\theta, \phi) \\ f_{4r}^{(1)}(\theta, \phi) & f_{4r}^{(2)}(\theta, \phi) & f_{4r}^{(3)}(\theta, \phi) \\ f_{4\theta}^{(1)}(\theta, \phi) & f_{4\theta}^{(2)}(\theta, \phi) & f_{4\theta}^{(3)}(\theta, \phi) \\ f_{4\phi}^{(1)}(\theta, \phi) & f_{4\phi}^{(2)}(\theta, \phi) & f_{4\phi}^{(3)}(\theta, \phi) \end{bmatrix},$$

$$\left[\left(\frac{r}{L}\right)^{-\lambda}\right] = \begin{bmatrix} \left(\frac{r}{L}\right)^{-\lambda_1} & 0 & 0 \\ 0 & \left(\frac{r}{L}\right)^{-\lambda_2} & 0 \\ 0 & 0 & \left(\frac{r}{L}\right)^{-\lambda_3} \end{bmatrix},\ [\boldsymbol{K}] = \begin{Bmatrix} K_1 \\ K_2 \\ K_3 \end{Bmatrix}$$

2.2. Eigen analysis formulated by FEM for calculation the order of stress singularity and the angular functions

The order of stress singularity, λ_n, is determined using the eigen analysis formulated by FEM (refer to Pageau and Biggers (1995) for details of the eigenvalue and eigenvector analysis in three-dimensional joints; and refer to Islam and Koguchi (2010) for piezoelectric material bonded joints).

The eigen equation derived by the principle of virtual work for calculating the eigenvalue, p, is expressed as follows:

$$(p^2[\mathbf{A}] + p[\mathbf{B}] + [\mathbf{C}])\{\mathbf{u}\} = 0, \tag{9}$$

where $[\mathbf{A}]$, $[\mathbf{B}]$, and $[\mathbf{C}]$ are matrices composed of material properties, $p = 1 - \lambda$, and $\{\mathbf{u}\}$ is the eigenvector of displacement and electric potential.