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Exact solutions for the effective nonlinear viscoelastic (or elasto-viscoplastic) behaviour of particulate composites under isotropic loading

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ABSTRACT

We consider a composite sphere which consists of a spherical inclusion embedded in a concentric spherical matrix, the inclusion and matrix phases obeying an isotropic nonlinear viscoelastic behaviour. For different isotropic loadings (macroscopic stress or dilatation, swelling of the inclusion phase), the general solutions are shown to depend on the shear stress distribution in the matrix. This shear stress distribution is solution of a first-order nonlinear integro-differential equation, regardless of the inclusion viscoplastic behaviour. When the viscous strain rate potentiel in the matrix is a power-law function of the von Mises equivalent stress, closed-form solutions are given for some special cases clearly identified. Full-field calculations of representative volume elements of particulate composites are also reported. For a moderate volume fraction of inclusions, the composite sphere model turns out to be in excellent agreement with these full-field calculations.

1. Introduction

Many technological materials are particulate composites (particles embeded in a matrix) in which the phases may undergo different stress-free strains (thermal dilatation, physico-chemical evolution, phase transformation, irradiation effect, …). This differential deformation of the phases induces internal stresses in the material, even for materials which are homogeneous elastically. If relaxation mechanisms like viscous (or viscoplastic) strains appear in the phases (effect of time, temperature, irradiation, …), this internal elastic stress field will relax heterogeneously. The internal stresses level as well as their time evolutions have to be known to assess the mechanical integrity of the considered composite. Thats why, even for particular situations, it would be of great interest to derive closed-form expressions of the time evolutions of stresses and strains of a nonlinear viscoelastic (or elasto-viscoplastic) composite submitted to differential stress-free strains. This is precisely the goal of this paper to yield analytical results and hence to allow a better understanding of the local distribution and time-evolution of the mechanical fields inside these particulate composites. Otherwise, as explained hereafter, these closed-form expressions will provide a reference solution to challenge homogenization methods for a class of particular microstructure (the composite sphere assemblage).

The particulate composites of interest in this study have a small or moderate volume fraction of inclusions (less than 30%) and therefore the interactions between inclusions are weak. In this case, classically, it is idealized by considering spherical inclusions with a gradation in size (composite sphere assemblage (Hashin, 1962)) and such that the ratio of the inclusions and matrix radi remain constant for all size of inclusions (self-similar spheres). Moreover, the distribution of inclusions is such that a volume filling configuration is obtained. Following (Hashin, 1962), the effective behaviour of such a microstructure is well approached by that of a composite sphere with the same volume fractions of the phases. The analytical results mentioned above are obtained by solving the mechanical problem on this simplified volume element.

For general microstructures, for which only some statistical informations are available, when the constituents display a linear elastic behaviour, homogenization methods provide bounds or estimates of the effective modulus (or compliance) tensor. For instance, the "Self-consistent" model (Kroner, 1978) provides reliable estimates for polycristalline microstructures while the "Mori-Tanaka" one (Mori and Tanaka, 1973) is well-suited to particle reinforced composites with a low volume fraction of particles.

With the help of the so called correspondence principle (Mandel, 1966), these models can be easily extend to linear viscoelastic composite materials to estimate the effective creep or relaxation functions. When (at least) one of the constituents of the composite material displays a nonlinear viscoelastic behaviour, several approaches have been proposed. The approach proposed by (Weng, 1981) consists in considering the viscous strains as homogeneous stress free strains, the associated first moment of the stress field in the phases being solutions of

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a linear thermoelastic problem. To improve these too stiff estimates, the "affine method" (Masson and Zaoui, 1999) is based on the solution of a linear thermo-viscoelastic problem. An alternative approach, using the correspondence principle and internal variables formulation was proposed by (Ricaud and Masson, 2009) in order to estimate the effective response of ageing viscoelastic composites.

More recently, variational approaches ((Suquet, 1995), (Ponte Castañeda and Suquet, 1998)) classically used for behaviours deriving from a single potential (nonlinear elasticity or viscosity) were extended by (Lahellec and Suquet, 2013) to nonlinear elastic-viscoplastic (and elastic-plastic) behaviours (which derive from two potentials). Based on the first and second moments of the fields, this approach has been shown to yield satisfactory estimates unlike "classical" mean-fields approaches based on the first moment of the fields.

Otherwise, analytical results obtained for particular microstructures are useful and it can be used to improve estimates derived by other homogenization methods. This is the case for the Eshelby's result (Eshelby, 1959), which gives the localization tensor of an ellipsoidal region embedded in an infinite linear elastic matrix. Eshelby's result is used in many others models which aim to estimate the effective behaviour of composites with a particle-matrix microstructure, with a low volume fraction of particles (as in the "Mori-Tanaka" model (Mori and Tanaka, 1973)).

Composite sphere volume element is extensively used in micromechanics due to the facility to derive analytical results for this simple microstructure. When the inclusions and the matrix obey a linear elastic behaviour, the effective properties of the composite can be bounded by solving the composite sphere problem: a spherical inclusion surrounded by a spherical layer of matrix (Hashin, 1962). For voids or rigid particles surrounded by a viscoplastic matrix, bounds have been derived by considering a similar problem. In (Michel and Suquet, 1992) or, more recently, in (Danas et al., 2008) the analytical results obtained for a hollow sphere have been used in order to improve variational bounds for viscoplastic porous media.

In porous plasticity, Gurson (1977) uses the hollow sphere with a von Mises matrix, to derive its well known criterion. Gurson analysis was extended to porous materials containing align ellipsoidal voids by (Gărăjeu, 1996), (Gologanu et al., 1993), (Flandi and Leblond, 2005) and (Madou and Leblond, 2012). Exact calculations by (Cazacu et al., 2013) carried out on a hollow sphere lead to a plasticity criterion which presents a third invariant effect. Additional results were obtained by (Le Quang and He, 2008) for elastic-plastic matrix and by (Thoré et al., 2009), (Anoukou et al., 2016), when the matrix phase is pressure-sensitive. In addition, the composite sphere problem has also been solved for elastic-plastic constituents behaviour (see (Chu and Hashin, 1971)). This solution is consistent with the particular one (when inclusions are voids) given by (Hill, 1998).

For linear viscoelasticity, estimates have been derived by using the composite sphere model and the correspondence principle (Christensen, 1969). Here, we aim at solving the composite sphere problem when the two phases display a nonlinear viscoelastic behaviour. To derive exact results, we will limit these analytical developments to isotropic loading. This isotropic loading will consist in a purely mechanical loading as dilatation or tension applied to the outer surface of the composite combined with a thermo-mechanical loading as a thermal strain mismatch between the two phases, appearing with temperature variations (thermomechanical loadings of structures, fabrication processes, etc.). The limit case of an incompressible matrix or of a composite sphere with homogeneous compressibility will be derived explicitly as well. The following theoretical developments are limited to infinitesimal strains ($\boldsymbol{\varepsilon}$ and $\boldsymbol{\sigma}$ denote the infinitesimal strain and Cauchy stress tensors, respectively).

The paper is organized as follows. In section 2, the composite sphere problem is formulated. The local fields in the inclusion and the equations for the local fields in the matrix are obtained in section 3. Section 4 presents the exact solutions obtained in the particular cases mentioned above. The predictions of the model are compared to the ones given by full-field computations of random particulate composites in section 5.

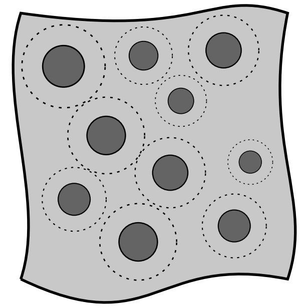

Fig. 1. The composite sphere (left); the composite spheres assemblage (right).

2. The composite sphere model

We consider a composite sphere (domain V) of external radius b containing in its center a non linear viscoelastic spherical inclusion of radius a (domain 2) (Fig. 1), embedded in a nonlinear viscoelastic matrix (domain 1). As shown by (Hashin, 1962), the effective elastic potential of such a simplified volume element is a bound for the elastic potential of more complex microstructures, obtained by spheres assemblage (Fig. 1).

In what follows indexes (1) and (2) are used for the mechanical fields and coefficients in the matrix and in the inclusion, respectively. Hence, $\chi_1(\boldsymbol{x})$ and $\chi_2(\boldsymbol{x})$ are the characteristic functions of domains 1 and 2, respectively, and the volume fractions of the matrix and of the inclusion are denoted by c_1 and c_2, $c_1 = \langle\chi_1(\boldsymbol{x})\rangle_V$, $c_2 = \langle\chi_2(\boldsymbol{x})\rangle_V = \frac{a^3}{b^3}$ and $c_1 + c_2 = 1$ ($\langle\ \rangle_V$ is the average operator on V).

The composite sphere is loaded isotropically on the outer boundary and by submitting a uniform isotropic stress-free strain to the inclusion. This stress-free strain may correspond to a thermal strain mismatch between the two phases. On the outer boundary, two kinds of loadings, which generalise relaxation and creep tests, are considered:

- homogeneous strain: $\boldsymbol{u}(\boldsymbol{x}, t) = E_m^0(t).\ \boldsymbol{x}$ for $\boldsymbol{x} \in \partial V$,
- homogeneous stress: $\boldsymbol{\sigma}(\boldsymbol{x}, t).\ \boldsymbol{n}(\boldsymbol{x}) = \Sigma_m^0(t).\ \boldsymbol{n}(\boldsymbol{x})$ for $\boldsymbol{x} \in \partial V$,

Assuming the small strain hypothesis, the decomposition of the total strain reads:

$$\boldsymbol{\varepsilon}(\boldsymbol{x}, t) = \boldsymbol{\varepsilon}^e(\boldsymbol{x}, t) + \boldsymbol{\varepsilon}^v(\boldsymbol{x}, t) + \varepsilon_0^{(2)}(t)\chi_2(\boldsymbol{x})\boldsymbol{\delta}, \tag{1}$$

where $\boldsymbol{\varepsilon}^e$ and $\boldsymbol{\varepsilon}^v$ are the elastic and viscous strains and $\varepsilon_0^{(2)}(t)\boldsymbol{\delta}$ is the uniform and spherical stress-free strain applied to the inclusion ($\boldsymbol{\delta}$ is the second order identity tensor).

The non linear viscous behaviour considered in this paper derives from a potential $w(\boldsymbol{x}, \boldsymbol{\sigma})$:

$$\dot{\boldsymbol{\varepsilon}}^v = \frac{\partial w}{\partial \boldsymbol{\sigma}} \text{ with}$$
$$w(\boldsymbol{x}, \underset{\sim}{\sigma}) = w_1(\underset{\sim}{\sigma})\chi_1(\boldsymbol{x}) + w_2(\underset{\sim}{\sigma})\chi_2(\boldsymbol{x}) \tag{2}$$

where w_1 and w_2 are the dissipation potentials (convex functions) of the matrix and of the inclusion respectively, which depend only on the square of equivalent stress $\underset{\sim}{\sigma} = \sigma_{eq}^2$. The mean and equivalent stresses σ_m and σ_{eq} are defined as usualy:

$$\sigma_m = \frac{1}{3}\mathrm{Tr}(\boldsymbol{\sigma}), \quad \boldsymbol{s} = \boldsymbol{\sigma} - \sigma_m\boldsymbol{\delta}, \quad \sigma_{eq} = \sqrt{\frac{3}{2}\boldsymbol{s}:\boldsymbol{s}}, \tag{3}$$

where $\boldsymbol{s}$ denotes the stress deviator.

Then, by derivation of (2) we get