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Scale effects in the hygro-thermo-mechanical response of fibrous networks

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ABSTRACT

The response of fibrous materials to complex mechanical, hygroscopic and thermal loadings is a relevant research question in many engineering fields. In the literature, the behaviour of fibrous networks is typically investigated by assuming uniform loading conditions at the micro-structural level and by developing homogenization schemes that provide structure-property relations. However, in a number of situations, for instance in paper media used in digital ink-jet printing applications, the length scale of the applied loading (in this case of the moisture distribution) may be comparable to the characteristic length scale of the micro-structure. Therefore, a homogenized description may not be adequate for capturing the response of the fibrous network, even in an average sense. Indeed, the response of the network may depend on the ratio between the typical length scale of the loading and that of the micro-structure. The goal of this paper is precisely to investigate the scale effect on the network response due to the application of hygroscopic (or thermal) loads, which may rapidly fluctuate over the micro-structure. To this aim, a two dimensional fibrous network model is exploited. This model properly represents several network level features, such as the fibre's hygro-elastic properties and geometry, orientation, areal coverage, etc. The model is subjected to different moisture profiles, ranging from slow to fast oscillations. This reveals a pronounced size effect in the network deformation: the faster the moisture fluctuation, the higher the network's average strain. By exploring the dependence of the size effect on different micro-structural parameters, it is shown that the size effect is strictly related to the accommodation of fibre swelling by the voided network regions, and that therefore it is governed by the average size of the voids.

1. Introduction

Fibrous materials are present in a large number of engineering applications, as for instance tissue, textiles, paper, ropes, non-woven networks. The effective response of fibrous networks is governed by several phenomena at different length scales: the behaviour of single fibres, their interaction in the bonding regions, their geometrical features and those of the network. This study is mainly motivated by cellulose-based fibrous networks, i.e. paper. Cellulose fibres are strongly sensitive to moisture content variations, which induce large anisotropic fibre swelling (up to 20%) and coupling phenomena between the network's mechanical and hygroscopic response (Niskanen, 1998; Larsson and Wagberg, 2008; Bosco et al., 2015a). Moisture induced deformations may lead to dimensional stability problems. These are generally manifested as in-plane or out-of-plane deformations, caused by the response of the network to inhomogeneous moisture content changes, both in the plane and through the thickness of the material. The main interest of this work is to focus on the effect of in-plane moisture gradients, and in particular to assess the relation between the response of the network and the applied macroscopic hygroscopic loads, when the scale of fluctuation of the latter becomes comparable to the characteristic size of the micro-structure (e.g. the fibre length). While this situation may have connections, for instance, with high-resolution digital ink-jet printing, where a single character/letter has often dimensions that are comparable to, or smaller than, the characteristic length of a single fibre (Le, 1998), the current study is mostly driven by scientific curiosity. In practical applications, through the thickness moisture gradients may be more critical in determining dimensional stability issues (Bosco et al., in press). Note finally that the following analysis focusses on the hygro-mechanical response of cellulose fibrous networks subjected to moisture content variations; however, the proposed methodology is general and can be used to investigate the response of a wider class of fibrous materials in the presence of complex hygroscopic, thermal, and possibly hygro-thermal loading conditions.

Several papers in the literature focus on the effective response of fibrous materials. Earlier models are based on analytical approaches, which often assume an affine deformation of the network, i.e. the

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macroscopic deformation and local strains are equal (Cox, 1952; Astrom et al., 1994; Wu and Dzenis, 2005 ; Tsarouchas and Markaki, 2011). These approaches provide closed-form expressions for the effective mechanical properties of the network as a function of microstructural features, such as fibre and network geometry, orientation and spatial distribution. Assuming affine deformation may however lead to an overestimation of the obtained effective properties (Hatami-Marbini and Picu, 2009). Therefore, several studies have proposed numerical or computational models of fibrous networks (Hatami-Marbini and Picu, 2009; Wilhelm and Frey, 2003; Bronkhorst, 2003; Hagglund and Isaksson, 2008; Isaksson and Hagglund, 2009; Kulachenko and Uesaka, 2012; Shahsavari and Picu, 2013; Shahsavari and Picu, 2015; Lu et al., 2014; Dirrenberger et al., 2014; Raina and Linder, 2014; Raina and Linder, 2015; Bosbach, 2015; Chen et al., 2016; Deogekar and Picu, 2017; Berkache et al., 2017). Most papers focus on two dimensional fibrous network representations in order to describe the network's mechanical behaviour. More recent work also includes the analysis of fibrous networks' hygro-thermo-mechanical effective response (Sellén and Isaksson, 2014; Bosco et al., 2017a, 2017b). Most of these studies are based on the assumption of a strong scale separation: the characteristic length over which the macroscopic load varies is much larger than the network's characteristic size, resulting in uniformly applied loads at the scale of the network. Only few of these analyses assess the effect on the network response of non-uniform loadings (Raina and Linder, 2014, 2015), and, to the best of the authors' knowledge, none of them focuses on non-uniform (in-plane) hygroscopic fluctuations.

The main goal of this paper is therefore to study how the effective response of (cellulose) fibrous networks scales with the scale of the applied moisture content variation, in the limit where this scale becomes comparable with the network's characteristic size. The proposed model departs from previous work (Bosco et al., 2017a; Bosco et al., 2017b), in which a periodic two dimensional network was subjected to a uniform hygro(thermal) load, and its effective properties were derived through asymptotic homogenization. The network is generated by adopting a random point field and an anisotropic orientation probability density function. The assumption of periodicity allows to consider the network as infinitely large. In order to describe the interplay in the bonding regions between the longitudinal and transverse hygroscopic and mechanical properties of the fibres, which drives the overall network hygro-expansive response, the fibres are modelled as two dimensional (transversely isotropic) elements. This is different from most work in the literature, where fibres are typically treated as one dimensional elements, either trusses or beams (e.g. Bronkhorst, 2003; Chen et al., 2016; Sellén and Isaksson, 2014; Strömbro and Gudmundson, 2008). In the present analysis, a hygro-elastic constitutive behaviour is adopted. Extensions towards irreversible material behaviour can be incorporated along the lines of Bosco et al., 2015b. The network is subjected to a non-uniform moisture content variation in the plane. It is assumed that the moisture penetrates instantaneously in the material and then no longer evolves, i.e. diffusion effects are neglected. A sinusoidal moisture distribution is adopted. This distribution is characterized by its wavelength, which is varied in a range from short (period much smaller than the fibre length) to long (period spanning multiple fibre lengths). Throughout the paper, long wavelength and short wavelength moisture profiles will be referred to as (space-wise) "slow" and "fast" moisture fluctuations, respectively. The influence of the wavelength of the moisture pattern relative to the network characteristic size on the overall network response is investigated, by performing finite element simulations on a set of network configurations. Additionally, the dependence of the scale effect on geometrical network parameters such as sparsity/density, fibre aspect ratio and anisotropy in the orientation, is analysed.

The paper is organized as follows. In Section 2, the network generation, the resulting boundary value problem, and the governing material parameters are discussed. Section 3 illustrates the size effect in the hygro-mechanical network's response, when subjected from slow to fast moisture fluctuations. The dependence on the network geometrical features is also discussed. Conclusions are finally given in Section 4.

2. The model

2.1. Network generation

The proposed fibrous network model is based on a two dimensional description. It refers to a coordinate system (x, y), with position vector $\mathbf{x} = x\mathbf{e}_x + y\mathbf{e}_y$, $\mathbf{e}_i$, $(i = x, y)$ being the unit vectors of a Cartesian vector basis. A square domain with edge L and area Q: $(0, L) \times (0, L)$ is considered, in which a set of seed points is randomly distributed. The seed points determine the location of the geometrical centres $\mathbf{x}_c$ of the fibres in the domain Q. To each seed point, a random orientation angle θ is associated, according to a wrapped Cauchy orientation distribution probability density function (Cox, 1952):

$$f(\theta) = \frac{1}{\pi}\frac{1 - q^2}{1 + q^2 - 2q\cos(2\theta)} \tag{1}$$

where $-\pi/2 < \theta \leq \pi/2$ is the angle between the fibre axis and the $x-$direction; $0 \leq q < 1$ is a measure of the orientation anisotropy.

The fibrous network is generated by depositing the fibres in the domain Q, based on the set of center points and orientations. Each individual fibre is represented as a two-dimensional, rectangular object with length l and width w. By trimming the portions of the fibres that fall outside the box Q along a certain edge and copying them into the domain at the opposite edge, a periodic network is generated. While the order of fibre deposition is not considered, the local number of fibres that overlap in a certain point of the domain and the corresponding orientations are stored for further calculation of the local hygro-mechanical properties. It is assumed that all fibres overlapping in a given point are perfectly bonded to each other, i.e. they are fully kinematically coupled. It may be remarked again that such a two dimensional fibre representation is not conventional in network models. It is however necessary to model the interplay between the hygro-expansive and mechanical response, on which the competition between the longitudinal and the transverse behaviour of a single fibre has a strong influence. Note finally that the regions of the plane that happen to be unfilled by fibres are essentially empty spaces through the thickness of the network; from now on they will be referred to as voids.

Due to the simplified planar description, it is not possible to define a true density for the network. An alternative measure for the "density"/"sparsity" of the system is given by introducing the notion of areal coverage $\bar{c}$, i.e. the ratio between the total area occupied by the fibres and the area of the domain Q:

$$\bar{c} = \frac{A_f}{L^2} = \frac{n_f wl}{L^2} \tag{2}$$

with the number of fibres in the network. The coverage can be interpreted as the average number of fibre layers characterizing the network and thus defines its average thickness; for this reason, it can take values exceeding unity.

The network is fully discretized by finite elements, following an efficient discretization strategy. The domain is first subdivided in a dense regular grid of square finite elements of edge length $l_e = w/\xi$ ($\xi \geq 1$, integer). The location of the geometrical center of each finite element e is used as the criterion to establish whether the element is part of a fibre, i.e. the geometrical center must be located inside the rectangular region of area $l \times w$ centred at $\mathbf{x}_c$ and oriented at the angle θ. This implies that the fibres have jagged edges. This affects only minimally the local stress and strain distributions if the discretization is sufficiently fine ($\xi \geq 5$), as reported in Bosco et al., 2017a.

Fig. 1 presents two examples of networks of coverage $\bar{c} = 1$, one with a uniform (isotropic) orientation distribution, $q = 0$ - Fig. 1(a), and one with an anisotropic orientation distribution, $q = 0.5$ - Fig. 1(b). Both networks are characterized by fibre length l, unit-cell edge length