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Axial shear fracture of a transversely isotropic piezoelectric quasicrystal cylinder: Which field (phonon or phason) has more contribution?

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ABSTRACT

Quasicrystals are a kind of new materials that exhibit a number of unique mechanical behaviors such as the coexistence of the phonon and phason fields. However, they are brittle and susceptible to the failure of fracture. As is known, both phonon and phason fields may contribute to fracture, but which one has more contribution? This is an open question that is of both scientific and engineering significance. The main purpose of the present paper is to answer this question. For this purpose, the axial shear fracture analysis is performed for a cylindrical composite that is made of 1D piezoelectric quasicrystals. Because of material symmetry (i.e. transverse isotropy), the problem is formulated in the cross section of the cylinder by using the polar coordinate system. The solution of the governing equation is expressed as an infinite series and the generalized dislocations are introduced to derive the singular integral equation, which is numerically solved. The computation is first verified by considering an existing example. Parametric studies then reveal that the fracture is dominated by the phason field when the phonon-phason loading ratio is below a critical point, but it is governed by the phonon field instead if the critical point is passed. In addition, the position of the critical point may be shifted by changing the values of a part of material properties only.

1. Introduction

Quasicrystals are a kind of new matters first discovered in the rapidly cooled Al-Mn alloys by Shechtman (see Shechtman et al., 1984). Since then, it has been successively confirmed by experiments that quasicrystals can occur in a number of other binary or ternary alloys (such as Al-Co, Al-Cr, Al-Cu-Fe, Ni-V, Ni-Ti, Cr-Ni-Si, etc) and even the piezoelectric ceramics (Ye et al., 1985; Hu et al., 1997). A unique feature of quasicrystals is that their atoms are quasiperiodically arranged in at least one direction. According to the number of quasiperiodicity directions, quasicrystals can be classified into three types, i.e., the 1D, 2D and 3D ones (Fan and Mai, 2004). Due to the quasiperiodic arrangement of atoms, quasicrystals have many particular novel properties that are generally not possessed by the conventional crystals. For example, their porosity, thermal conductivity, frictional coefficient and adhesion coefficient are very low, but their electric resistivity and abrasion resistance are quite high (Fan, 2011). Therefore, quasicrystals have wide application prospects in the industries of optics, electronics, communication, and so on (Dubois, 2005).

In the applications of quasicrystals, it is necessary to understand their mechanical behavior. For this reason, the theory of elasticity of quasicrystals has been an active research topic in the past thirty years. Due to the quasiperiodic arrangement of atoms, the 3D-space based theory for the conventional crystals cannot be directly applied in formulating quasicrystals (Levine et al., 1985; Ding et al., 1993). Because the quasiperiodic structures in the 3D physical space can be projected into a higher-dimensional super space as periodic structures, the theory of elasticity for the 1D, 2D and 3D quasicrystals is established in the 4D, 5D and 6D spaces (Fan, 2011), respectively. The mechanical fields in the original 3D space are called the phonon fields, while those in the complementary higher-dimensional space are called the phason fields. These two kinds of fields are generally coupled together. Therefore, the theory of elasticity of quasicrystals mainly formulates the phonon fields, phason fields and their coupling (Levine et al., 1985; Ding et al., 1993; Fan, 2011). Besides the phonon-phason coupling, quasicrystals may also exhibit the electro-phonon and electro-phason couplings. In order to consider them, the abovementioned elasticity theory has also been extended to describe the quasicrystal piezoelectric behavior (Hu

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et al., 1997; Altay and Dokmeci, 2012). Using these theories, researchers have solved a series of typical mechanical problems such as the dislocations, cracks, indentation, contact, bending, vibration and wave propagation, etc. The reader is referred to Yang et al. (2017a; 2017b; 2014), Zhao et al. (2016), Waksmanski et al. (2016), Agiasofitou and Lazar (2014), Li et al. (2014), Wang and Schiavone (2014), Sladek et al. (2013a), Gao (2009), Freedman et al. (2006), Chen et al. (2004), and the references cited therein.

Among the abovementioned mechanical problems, fracture is a main concern, because the inherent brittleness makes quasicrystals quite sensitive to cracks (Zhao et al., 2017; Dang et al., 2017). In fact, cracking is a typical failure mode and fracture analysis is very crucial to the optimal design of quasicrystal structures. In these years, fracture mechanics of quasicrystals has drawn more and more attention from the researchers. Li et al. (1999) first considered a Griffith crack in a decagonal quasicrystal and revealed that the crack-tip stress has the conventional square-root type singularity. Peng and Fan (2000) proposed the perturbation method for solving the fracture problem of a 3D icosahedral quasicrystal that contains a circular crack. Yin et al. (2002) derived the exact solution for a mode II crack in a 2D octagonal quasicrystal by the method of dual integral equations. Mariano et al. (2004) determined the distribution of phonon and phason displacements around the crack tip in Al–Pb–Mn quasicrystals and interpreted the phonon–phason coupling based on a stochastic aspect. Radi and Mariano (2010) applied the extended Stroh formalism in studying the problems of stationary straight cracks in quasicrystals. Guo and Lu (2011) considered four cracks originating from an elliptic hole in 1D hexagonal quasicrystals and obtained the exact solutions of the stress intensity factors. Fan et al. (2012) presented the linear, nonlinear and dynamic fracture theories for different kinds of quasicrystals. Sladek et al. (2013b) proposed a meshless local Petrov-Galerkin method for the fracture analysis of decagonal quasicrystals. Li (2013) presented the fundamental solutions of penny-shaped and half-infinite plane cracks embedded in an infinite space of 1D hexagonal quasi-crystal under thermal loading. Gao et al. (2014) developed the explicit solutions for the coupled fields around a hole in 3D quasicrystals and then derived the weight function for a crack. Li (2014) presented the solution of the elastic field around a planar crack in an infinite medium of 1D hexagonal quasicrystal, and Wang et al. (2015) solved the corresponding problem in 2D hexagonal quasicrystal. Tupholme (2015) investigated the problem of an anti-plane shear crack moving in 1D hexagonal quasicrystals and obtained the solution of the stress intensity factor. Yang and Li (2016) studied a circular hole with a straight crack in 1D hexagonal piezoelectric quasicrystals. Fan et al. (2016) developed the fundamental solutions of 3D cracks in 1D hexagonal piezoelectric quasicrystals. Li (2016) solved the problem of multiple collinear cracks in a 1D hexagonal quasicrystal strip with infinite length. Li et al. (2017) derived the 3D fundamental thermo-elastic field in an infinite space of 2D hexagonal quasi-crystal with a penny-shaped/half-infinite plane crack. Yang et al. (2017b) considered two electrically limited permeable cracks that emanate from an elliptic hole in 1D hexagonal piezoelectric quasicrystals and obtained the fracture parameters by using the conformal mapping and Stroh-type formalism. Zhao et al. (2017) and Dang et al. (2017) analyzed the 3D arbitrarily shaped interfacial cracks in a 1D hexagonal thermo-electro-elastic quasicrystal bi-material. Wang and Ricoeur (2017) made a numerical path prediction for a crack in 1D quasicrystals under mixed-mode loading. Tupholme (2017) derived the analytical expressions for an embedded crack moving in 1D piezoelectric quasicrystals that is subjected to non-uniform loading. Cheng et al. (2017) solved the fracture problem of a finite rectangular quasicrystal plate by the boundary collocation method. Although so much research has been done in this field, most papers only dealt with cracks in infinite or semi-infinite quasicrystals. Up till now, the fracture of finite quasicrystal structures has been scarcely investigated.

Finally, as stated above, quasicrystals have two kinds of mechanical fields, one being phononic and the other phasonic. Both may contribute to fracture, because cracking is essentially a mechanical process. However, which field (phonon or phason) makes the main contribution? This question has not been clarified in existing papers, but it is scientifically important for better understanding the fracture mechanism of quasicrystals and meanwhile significant for the anti-fracture design of quasicrystal structures. Aiming at answering this question, we consider the axial shear fracture problem of a piezoelectric quasicrystal cylinder in the present work. The solutions of the fracture parameters are obtained by the method of singular integral equation and verified in a special case. The main contributor to the fracture and the related preconditions are discussed based on the numerical results.

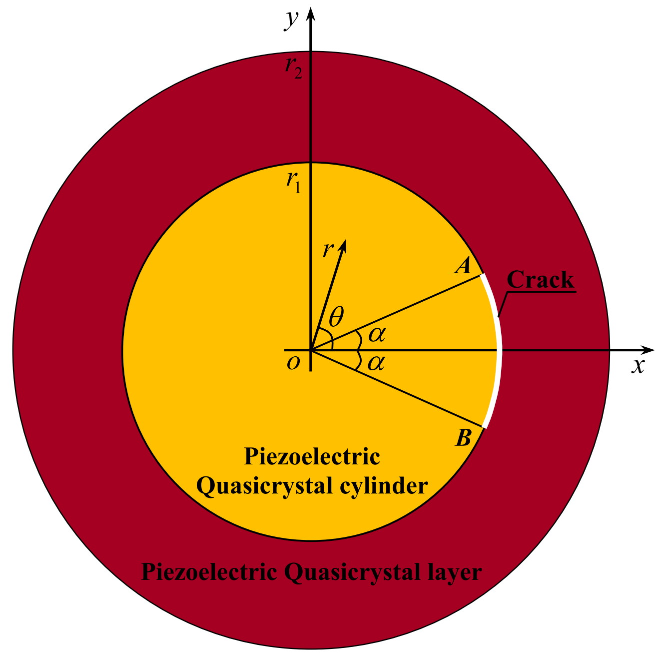

Fig. 1. Fracture model of a cylindrical piezoelectric quasicrystal composite.

2. Problem formulation

Fig. 1 shows the fracture model of a piezoelectric quasicrystal cylindrical composite. It consists of a central cylinder and an outer layer, whose inner and outer radii are r_1 and r_2. The interface is partially debonded and the central angle of the interfacial crack is 2α. A Cartesian coordinate system is set up in such a way that the origin coincides with the center of the cylinder, the rightward x-axis passes the center of the crack, the y-axis points upwards and the z-axis is determined by the right-hand rule. A polar coordinate system is also set up with the circumferential coordinate θ going anticlockwise from the positive x-axis.

The composite is polarized and quasi-periodic along the z-axis. This means that it is isotropic in its cross section. Assume that the composite is under axial shear. Then, the present fracture problem belongs to the anti-plane type. In this case, the basic equations have the form

$$\boldsymbol{\tau}_k^{(j)} = r^{-\delta_{k\theta}}\mathbf{M}^{(j)}\mathbf{u}_{,k}^{(j)},\ (k = r,\ \theta;\ j = 1,\ \ 2), \tag{1}$$

$$r\,\boldsymbol{\tau}_{r,r}^{(j)} + \boldsymbol{\tau}_{\theta,\theta}^{(j)} + \boldsymbol{\tau}_r^{(j)} = 0,\ (j = 1,\ \ 2), \tag{2}$$

where $\delta_{k\theta}$ is the Kronecker symbol. $\boldsymbol{\tau}_k^{(j)} = \{\tau_{zk}^{(j)}\ \ H_{zk}^{(j)}\ \ D_k^{(j)}\}^{\mathrm{T}}$ and $\mathbf{u}^{(j)} = \{u_3^{(j)}\ \ w^{(j)}\ \ \varphi^{(j)}\}^{\mathrm{T}}$. $\tau_{zk}^{(j)}$ and $H_{zk}^{(j)}$ are the phonon and phason stress components. $u_3^{(j)}$ and $w^{(j)}$ are the axial phonon and phason displacement components. $D_k^{(j)}$ and $\varphi^{(j)}$ are the electric displacement component and electric potential. $\mathbf{M}^{(j)}$ is the material property matrix that is given in the appendix.

In the present work, the comma followed by coordinates in the subscripts denotes partial derivatives. Superscripts 1 and 2 are used to