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Assessment of porosity influence on vibration and static behaviour of functionally graded magneto-electro-elastic plate: A finite element study

M.C. Kiran, S.C. Kattimani*

Department of Mechanical Engineering, National Institute of Technology Karnataka, Surathkal 575025, India

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ABSTRACT

In this paper, the free vibration characteristics and the static behaviour of porous functionally graded magneto-electro-elastic (FGMEE) plate is investigated using finite element method. The porosities arise due to the maladies in the fabrication processes and such porosities or micro-voids are accounted using modified power law. Influence of different porosity distributions on the behaviour of PFGMEE plate are considered in this study. The through thickness variation of material properties is achieved to obtain a functionally graded MEE plate. The coupled constitutive equations along with the principle of virtual work are used to develop a FE model for FGMEE plates. Influence of various porosity distributions on the structural behaviour of the plate is thoroughly investigated. The effect of porosity volume and material gradient index on the free vibration and static behaviour is explicitly studied. This study also includes the evaluation of the effect of geometrical parameters such as thickness ratio, aspect ratio, and boundary condition on the structural characteristics of porous FGMEE plate.

1. Introduction

Magneto-electro-elastic (MEE) materials are the combination of piezoelectric, Barium Titanate ($BaTiO_3$) and magnetostrictive, Cobalt Ferrite ($CoFe_2O_4$) materials. Such materials exhibit magneto-electric coupling which is absent in their individual phases (Boomgaard and Born, 1978). This unique property facilitates MEE materials to be largely sought in sensors and actuators applications. MEE composites exist in layered, multiphase, and functionally graded forms (Buchanan, 2004). Multilayered MEE plates are extensively investigated to assess their free vibration characteristics, buckling, and static behaviour under various loading conditions (Kiran and Kattimani, 2017, 2018a; Ramirez et al., 2006; Chen et al., 2014; Lage et al., 2004; Simoes Moita et al., 2009). Pan and his co-researchers (Pan, 2001; Pan and Heyliger, 2003; Pan and Han, 2005) proposed various analytical solutions to evaluate free vibration and static response of MEE plate. The behavioural study of MEE plate for free vibration and large deflection was established by Millazo (Milazzo, 2014a, 2014b, 2016; Kattimani and Ray, 2014a) via various methodologies. Kattimani and Ray, 2014a, 2014b discussed active constrained layered damping as an effective measure to control non-linear vibrations in MEE plates and shells. The scaled boundary FE method was implemented by Liu et al. (2016) to ascertain the higher order solutions for MEE plate composed of non-uniform material. Wakmanski and Pan (Waksmanski and Pan, 2016) evaluated free vibration of multilayered MEE plate with non local effect using 3-D analytical solutions. To reduce or eliminate the interface stresses existing in laminated composites, functionally graded materials were developed. The FG material properties vary throughout the thickness. The presence of functionally graded material in various applications has been increasing with the innovation in cutting edge manufacturing techniques (Mortensen and Suresh, 1995; Pompe et al., 2003; Miyamoto et al., 2013). The various structural characteristics of FGMEE material have been explicitly studied by many researchers (Ebrahimi et al., 2009; Ebrahimi and Rastgoo, 2009, 2011; Vinyas and Kattimani, 2017). Kattimani and Ray (2015) researched large-amplitude vibration responses of FG MEE plates. Recently, Kiran and Kattimani (2018b) investigated the frequency and static characteristics of skew-FGMEE plate.

The recent development in FGM includes the graded porosity structures. The pores in the microstructures of such structural materials are accounted via local density of the material. The methods to prepare FGMs are a trending area of research capturing attention of many researchers. The preparation method includes powder metallurgy, vapour deposition, self propagation, centrifugal casting, and magnetic separation (Khor and Gu, 2000; Barati, 2018; Watanabe et al., 2001; Song et al., 2007; Peng et al., 2007). Although many preparation methods are available, the sintering process is preferred due to its cost effectiveness. The FGMs prepared using sintering process possesses micro-voids or porosities due to the different solidification rate of material constituents (Zhu et al., 2001). A study by Wattanasakulpong et al. (2012) projects

* Corresponding author.
E-mail address: sck@nitk.ac.in (S.C. Kattimani).

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the importance of considering porosity factor in the design and analysis of FGMs. Wang et al. (2017) investigated the vibration characteristics of FG plates with porosities. Recently, Kiran and Kattimani (2018c) investigated the influence of porosities on the skew FGMEE plate. Ebrahimi et al. (2017a) analysed the vibration characteristics of MEE heterogeneous porous material plates resting on elastic foundations. Aero-hygro-thermal stability analysis of higher-order refined supersonic FGM panels with even and uneven porosity distributions was studied by Barati and Shahverdi (2017). Using refined four-variable theory, Barati et al. (2017) studied the electro-mechanical vibration of smart piezoelectric FG plates with porosities. Ebrahimi et al. (2017b) studied the free vibration of smart porous plates subjected to various physical fields considering neutral surface position.

Though, the recent developments in manufacturing techniques have improved significantly, the porosity is a common defect often observed in FGMs. Hence, it is intended to develop a suitable FE model to study the behaviour of FGMEE plates accounting the inherent porosity in the material. Studies on static analysis and free vibration characteristics of $BaTiO_3$–$CoFe_2O_4$ plates with porosity distribution are scarce in the literature. Hence, in this article, the finite element formulation to evaluate the free vibration and static characteristics of porous FGMEE plate for different porosity models is considered for evaluation. The effect of different porosity distribution, porosity volume index, and gradient index affecting the structural behaviour of porous FGMEE plate is extensively investigated. Further, the effect of thickness ratio, aspect ratio, and boundary condition is studied.

2. Problem description and governing equation

A schematic diagram of a porous functionally graded magneto-electro-elastic (FGMEE) plate with a Cartesian coordinate system attached to the corner of the plate is shown in Fig. 1. The length, the width and the total thickness of the plate are a, b and h, respectively. The material properties of the porous FGMEE plate are assumed to vary across the thickness. The bottom surface of the plate is piezoelectric ($BaTiO_3$) and the top surface being magnetostrictive ($CoFe_2O_4$). The plate model involved in the present analysis is developed by Hilderbrand et al. (Hildebrand et al., 1949). The displacement components u, v and w along x-, y-, and z-direction at any point in the porous FGMEE plate can be represented by (Hildebrand et al., 1949)

$$\begin{aligned} u(x,y,z,t) &= u_0(x,y,t) + z\theta_x(x,y,t) \\ v(x,y,z,t) &= v_0(x,y,t) + z\theta_y(x,y,t) \\ w(x,y,z,t) &= w_0(x,y,t) + z\theta_z(x,y,t) + z^2\kappa_z(x,y,t) \end{aligned} \tag{1}$$

where, u_0 and v_0 are the translational displacements at any point on the mid-plane of the plate along x- and y-directions while w_0 is the transverse displacement along z-direction at any point in the porous FGMEE plate. θ_x denote the generalized rotation of the normal to the middle plane of the porous FGMEE plate about the y-axis while θ_y denote the generalized rotation of the normal to the middle plane of the porous FGMEE plate about the x - axis. θ_z and κ_z are the generalized rotational displacements for the porous FGMEE plate with respect to the thickness coordinate. For the ease of computation, rotational and translational displacements are considered separately as follows:

$$\{d_t\} = [u_0\ v_0\ w_0]^T \{d_r\} = [\theta_x\ \theta_y\ \theta_z\ \kappa_z]^T \tag{2}$$

The shear locking in the thin structures is overcome by employing the selective integration rule and also facilitates the computation of elemental stiffness matrices linked with the transverse shear deformation in detail. This specific need is achieved by considering the state of strain at any point in the plate, separated by in-plane and transverse normal strain vector $\{\varepsilon_b\}$ and the transverse shear strain vector $\{\varepsilon_s\}$ given as

$$\{\varepsilon_b\} = [\varepsilon_x\ \varepsilon_y\ \varepsilon_z\ \nu_{xy}]^T \{\varepsilon_s\} = [\nu_{xz}\ \nu_{yz}]^T \tag{3}$$

where, ε_x, ε_y and ε_z represent the normal strains along x-, y- and z-directions, respectively; ν_{xy} represents the in-plane shear strain, ν_{xz} and ν_{yz} are the transverse or out of plane shear strains. Making use of the displacement field given in Eq. (1) and from the linear strain-displacement relations, the strain vectors $\{\varepsilon_b\}$ and $\{\varepsilon_s\}$ defining the state of inplane, transverse normal and transverse shear strain at any point in the porous FGMEE plate can be expressed as

$$\{\varepsilon_b\} = \{\varepsilon_{bt}\} + [Z_1]\{\varepsilon_{rb}\}\{\varepsilon_s\} = \{\varepsilon_{ts}\} + [Z_2]\{\varepsilon_{rs}\} \tag{4}$$

wherein the transformation matrices $[Z_1]$ and $[Z_2]$ are expressed as

$$[Z_1] = \begin{bmatrix} z & 0 & 0 & 0 & 0 \\ 0 & z & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 2z \\ 0 & 0 & z & 0 & 0 \end{bmatrix} [Z_2] = \begin{bmatrix} 1 & 0 & z & 0 & z^2 & 0 \\ 0 & 1 & 0 & z & 0 & z^2 \end{bmatrix}$$

The generalized strain vectors appearing in Eq. (4) are given by

$$\{\varepsilon_{bt}\} = \left[\frac{\partial u_0}{\partial x}\ \frac{\partial v_0}{\partial y}\ 0\ \frac{\partial u_0}{\partial y} + \frac{\partial v_0}{\partial x}\right]\{\varepsilon_{ts}\} = \left[\frac{\partial w_0}{\partial x}\ \frac{\partial w_0}{\partial y}\right]$$

$$\{\varepsilon_{rb}\} = \left[\frac{\partial \theta_x}{\partial x}\ \frac{\partial \theta_y}{\partial y}\ \frac{\partial \theta_x}{\partial x} + \frac{\partial v_0}{\partial x}\ \theta_z\ \kappa_z\right] \text{and} \{\varepsilon_{rs}\} = \left[\theta_x\ \theta_y\ \frac{\partial \theta_z}{\partial x}\ \frac{\partial \theta_z}{\partial y}\ \frac{\partial \kappa_z}{\partial x}\ \frac{\partial \kappa_z}{\partial y}\right]^T$$

Analogous to the strain vectors given in Eq. (3), the state of stress at any point in the porous FGMEE plate can be written as follows:

$$\{\boldsymbol{\sigma}_b\} = [\sigma_x\ \sigma_y\ \sigma_{xy}\ \sigma_z]^T \{\boldsymbol{\sigma}_s\} = [\tau_{xz}\ \tau_{yz}]^T \tag{5}$$

in which, σ_x, σ_y and σ_z are the normal stresses along x-, y- and z-directions, respectively; σ_{xy} is the in-plane shear stress; τ_{xz} and τ_{yz} are the transverse shear stresses along xz- and yz-directions, respectively. Considering the effect of coupled fields, the constitutive equations for the porous FGMEE plate can be expressed as follows:

$$\{\sigma_b\} = [\overline{C}_b(z)]\{\varepsilon_b\} - \{e_b(z)\}E_z - \{q_b(z)\}H_z\{\boldsymbol{\sigma}_s\} = [\overline{C}_s(z)]\{\varepsilon_s\} \tag{6a}$$

$$D_z = \{e_b(z)\}^T\{\varepsilon_b\} + \xi_{33}(z)E_z + d_{33}(z)H_z \tag{6b}$$

$$B_z = \{q_b(z)\}^T\{\varepsilon_b\} + d_{33}(z)E_z + \mu_{33}(z)H_z \tag{6c}$$

where, $[\overline{C}_b(z)]$ and $[\overline{C}_s(z)]$ are the functionally graded material coefficient matrices given as $[\overline{C}_b(z)] = \begin{bmatrix} \overline{C}_{11}(z) & \overline{C}_{12}(z) & \overline{C}_{13}(z) & \overline{C}_{16}(z) \\ \overline{C}_{12}(z) & \overline{C}_{22}(z) & \overline{C}_{23}(z) & \overline{C}_{26}(z) \\ \overline{C}_{13}(z) & \overline{C}_{23}(z) & \overline{C}_{33}(z) & \overline{C}_{36}(z) \\ \overline{C}_{16}(z) & \overline{C}_{26}(z) & \overline{C}_{36}(z) & \overline{C}_{66}(z) \end{bmatrix}$

$[\overline{C}_s(z)] = \begin{bmatrix} \overline{C}_{55}(z) & \overline{C}_{45}(z) \\ \overline{C}_{45}(z) & \overline{C}_{44}(z) \end{bmatrix}$ (7).While, $\xi_{33}(z)$ and $\mu_{33}(z)$ are the dielectric

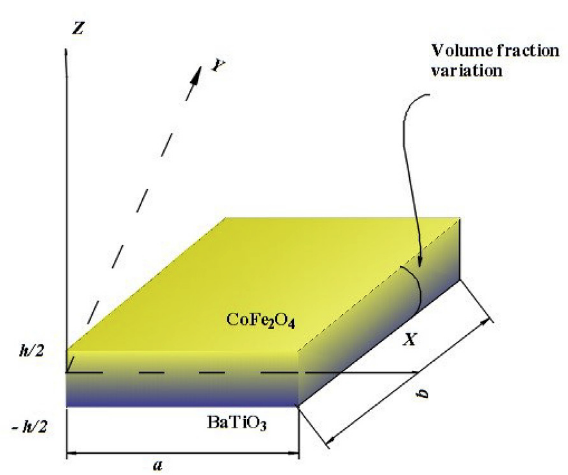

Fig. 1. Functionally graded MEE plate.