Form II of Mindlin's second strain gradient theory of elasticity with a simplification: For materials and structures from nano-to macro-scales

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Form II of Mindlin's second strain gradient theory of elasticity with a simplification: for materials and structures from nano- to macro-scales

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Abstract

The fundamental equations for Form II of Mindlin's second strain gradient elasticity theory for isotropic materials are first derived. A corresponding simplified formulation is then proposed, with six and two higher-order material parameters for the strain and kinetic energy, respectively. This simplified model is still capable of accounting for free surface effects and surface tension arising in second strain gradient continua. Within the simplified model, at first, surface tension effects appearing in nano-scale solids near free boundaries are analyzed. Next, a thin strip under tension and shear is considered and closed-form solutions are provided for analyzing the free surface effects. Expressions for effective Poisson's ratio and effective shear modulus are proposed and found to be size-dependent. Most importantly, for each model problem a stability analysis is accomplished disallowing non-physical solutions (befallen but not exclusively disputed in a recent Form I article). Finally, a triangular macro-scale lattice structure of trusses is shown, as a mechanical metamaterial, to behave as a second strain gradient continuum. In particular, it is shown that initial stresses prescribed on boundaries can be associated to one of the higher-order material parameters, modulus of cohesion, giving rise to surface tension. For completeness, a numerical free vibration eigenvalue analysis is accomplished for both a fine-scale lattice model and the corresponding second-order continuum via standard and isogeometric finite element simulations, respectively, completing the calibration procedure for the higher-order material parameters. The eigenvalue analysis confirms the necessity of the second velocity gradient terms in the kinetic energy density.

Keywords: second strain gradient elasticity, third displacement gradient elasticity, stability analysis, surface effects, surface tension, size effects, nano-structures, mechanical metamaterials, architectured materials, lattice structures, effective material moduli, dispersion relation

1 Introduction

Material modelling, in its broad meaning, is a fundamental task to be accomplished in order to adequately describe and predict the mechanical behaviour of solids. Although all materials are known to have a discrete nature, real physical systems can be modelled by the classical theories of continuum mechanics introduced and at first developed by such famous names as Piola, Poisson, Navier, Cauchy, and many others. The classical continuum models assume that the characteristic length scale of the material, e.g., grain or inhomogeneity size, is much smaller than the representative volume of the underlying averaging principles. This assumption is not necessarily valid, however, when modelling, on one hand, micro- and nano-scale objects such as MEMS or NEMS, or on the other hand, meso- and macro-scale discrete systems such as mechanical metamaterials or lattice structures. At nano-scale, due to the increasing surface-to-volume ratio, most materials demonstrate a very strong size-dependent behaviour. As reported in many works (see [1] and [2], for instance), atoms lying near a free surface are in the presence of different bindings than atoms in the bulk due to a redistribution of the underlying