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Response of an infinite beam on a bilinear elastic foundation: Bridging the gap between the Winkler and tensionless foundation models

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ABSTRACT

The response of an infinite beam on an elastic foundation depends on the property/modeling of the foundation. There is a qualitative difference between the responses of a beam when it is on the Winkler foundation and on the tensionless foundation. A bilinear elastic foundation model, which describes the different behaviors of the elastic foundation in the tensile and compressive zones with two different foundation moduli, is proposed and a straightforward computational method is also formulated. The Winkler and tensionless foundations are shown to be the two special cases of the bilinear elastic foundation model. With this bilinear elastic foundation model, a more general method of modeling an elastic foundation is provided, which can be of some help to the modeling of the support of the ballastless high-speed railway.

1. Introduction

Fig. 1(a) shows an infinite beam on an elastic foundation subjected to a concentrated load, which, for many years, has been an important model to analyze the deflections and stresses of a railroad track (Ang and Dai, 2013; Bian et al., 2014; Choros and Adams, 1979; Hetényi, 1946; Kerr, 1972, 1974; 1976; Lancioni and Lenci, 2010; Lin and Adams, 1987; Tran et al., 2014). In a regular ballasted railroad, the rail-tie frame is laid on a layer of crushed stone called ballast (Kerr, 1974). The rail actually lifts off its ballast in front of, and behind a moving train (Choros and Adams, 1979; Lin and Adams, 1987). Once the lift-off occurs, the ballast layer as an elastic foundation cannot exert any (tensile) force on the rail (Choros and Adams, 1979; Lin and Adams, 1987). While, the Winkler foundation model assumes that there is only one foundation modulus, which physically means that the Winkler foundation reacts the same in tension as in compression. Although nowadays the Winkler foundation model is still widely used to analyze the rail system (Ang and Dai, 2013; Tran et al., 2014), its usage is motivated more by the desire for mathematical simplicity than by physical reality (Lin and Adams, 1987). Therefore, the tensionless foundation model is more appropriate and often applied to study the contact between the rail and ballast (Choros and Adams, 1979; Lancioni and Lenci, 2010; Lin and Adams, 1987). The tensionless foundation is also called the foundation that reacts in compression only (Weitsman, 1970) or the unilateral springs/supports/foundation (Bhattiprolu et al., 2013, 2014; 2016; Dempsey et al., 1984; Lancioni and Lenci, 2010). Because there is no bonding force between a structure and the tensionless foundation, their contact is referred to as the unbonded contact (Weitsman, 1969); because a structure can separate/lift off the tensionless foundation, which results in a decreasing contact area, their contact is also referred to as the receding contact (Keer et al., 1972). However, when the tensionless foundation model is applied to the ballastless track system, which has been extensively used in the high-speed railway, a serious problem may arise: The ballastless railway support can take some tension (Bian et al., 2014). A ballastless high-speed railway consists of the following two parts (Bian et al., 2014): (1) the track superstructure (rail, fastener, track slab, cement asphalt mortar (CAM) layer and concrete base) and (2) the geotechnical substructure (roadbed, subgrade and subsoil). A major difference between the ballastless and ballasted tracks is the concrete track slabs replacing the ballast layer (Bian et al., 2014). The fasteners tightly bond the rail with track slab and the CAM layer bonds the track slab with the concrete base, which makes the rail separation from its support extremely difficult if there is any. Furthermore, the support capability of bearing tensile stress may become substantial because of the tight bondings between the rail/track slab, track slab/CAM and CAM/concrete base, etc. Actually, tensile stress was indeed detected by the sensors embedded in the roadbed layer underneath the concrete base (Bian et al., 2014). Recent elasticity analyses show that even for a homogeneous elastic continuum modeled as an elastic half-space, its surface responses to tension and compression are intrinsically different (Zhuo and Zhang, 2015a, 2015b). This asymmetric response to tension and compression is

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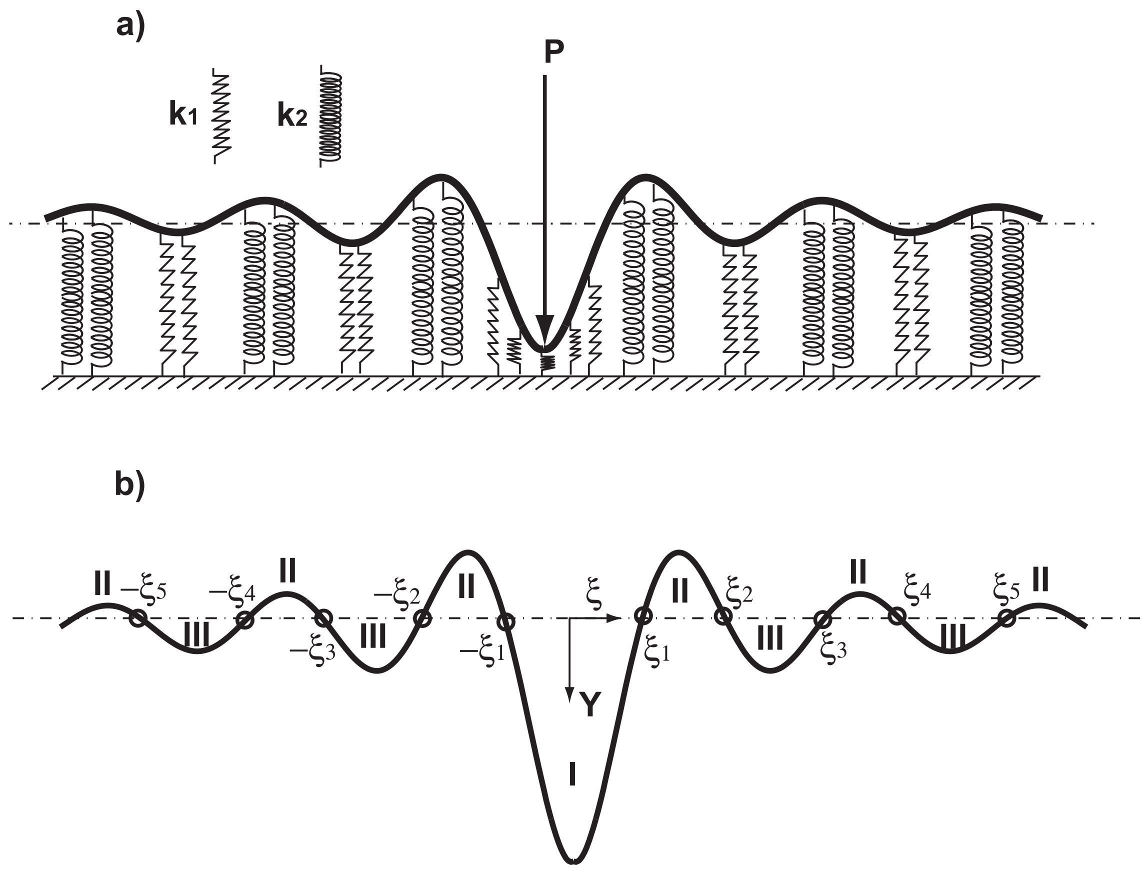

Fig. 1. (a) Schematic diagram of an infinite beam on a bilinear elastic foundation subjected to a concentrated load P. When the beam bends downwards/upwards, the foundation is with compression/tension and the corresponding foundation modulus is k_1/k_2. When $k_2 = 0$, the foundation is the tensionless foundation; when $k_2 = k_1$, it is the Winkler foundation. (b) The coordinate system and deflection areas. The zero points (ξ_i), which demarcate the areas, are marked with circles. Area **I** is the downward compressive deflection area with the concentrated load P and in comparison, area **III** is the other downward compressive deflection areas with no P. Area **II** are the upward tensile deflection areas.

also the mechanism responsible for the period-doubling behavior of the perfectly bonded film/substrate system in a post-buckling region (Brau et al., 2011). A bilinear elastic foundation model with two different foundation moduli of k_1 and k_2 as shown in Fig. 1(a) is thus proposed to model the different behaviors in the tensile and compressive zones. The bilinear behavior of the ballastless railway support is expected because of its heterogeneous multilayer structure and the interface properties. In this bilinear model, the Winkler foundation and the tensionless foundation are the two special cases of $k_2/k_1 = 1$ and $k_2/k_1 = 0$, respectively.

The contact mechanics of a beam on the tensionless foundation has been intensively studied for many years and various solution methods have also been developed. Although the Lagrange multiplier or penalty-based algorithms in several finite element analysis (FEA) commercial softwares are capable of modeling the unilateral constraint, the computational costs are very expensive in both storage and CPU time (Silveira et al., 2008). Furthermore, the complexity of the tools required to perform a comprehensive study or analyses such as design optimization, feedback control or stability etc can be infeasible for a full-scale FEA simulation (Attar et al., 2016; Silveira et al., 2008). There have always been interests to develope the methods to reduce the computational cost in an extensive parametric study of tensionless contact (Attar et al., 2016). A major difficulty in the tensionless contact problem is the unknown property of contact area, which makes the problem nonlinear and extremely difficult to be solved in some scenarios. Therefore, recent studies have been focusing on developing efficient methods of determining the contact area (Attar et al., 2016; Bhattiprolu et al., 2013, 2014; 2016; Ma et al., 2009a, 2009b; 2011; Nobili, 2013; Silveira et al., 2008). The incremental-iterative methods (Attar et al., 2016; Bhattiprolu et al., 2013; Silveira et al., 2008) update the governing equation/stiffness matrices in each iteration by tracking the structure deflections and continue until the convergence, which can still involve significant computation efforts. For example, a large number of modes (up to 20) in the Galerkin method are required to achieve a satisfying accuracy (Bhattiprolu et al., 2013). The transfer displacement function method (Ma et al., 2009a) reduces the computation by solving the contact/noncontact zones and deflections one by one. Besides, the tensionless contact properties can also be utilized to reduce the computational efforts. For example, there is an outstanding characteristics in the beam tensionless contact subjected to a concentrated load: Only one contact area exists for both the infinite (Tsai and Westmann, 1967; Weitsman, 1970, 1972) and finite (Zhang, 2008; Zhang and Murphy, 2004, 2013) beams. Even in the contact dynamics with a moving concentrated load, the one contact area conclusion still holds as far as the moving speed of the concentrated load is less than the critical speed of $\sqrt[4]{4EIk_1/m^2}$ (EI, m are the beam bending stiffness and mass per unit length) (Weitsman, 1971). It needs to keep in mind that under complex loads, the scenario of multiple contact areas can occur (Ma et al., 2009a, 2009b; 2011; Nobili, 2013). The zero points as called by Hetényi (1946) and shown in Fig. 1(b) are the points at which the beam deflection is zero. For the tensionless contact, the zero points are also the lift-off/separation points demarcating the contact areas (Weitsman, 1970). For an infinite beam, by reducing the two zero points to one via the symmetry property, the analytical (Weitsman, 1970) or approximate analytical solution (Weitsman, 1972) can be derived.

Compared with the abundant literature of the tensionless contact,