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A failure criterion for homogeneous and isotropic materials distinguishing the different effects of hydrostatic tension and compression

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ABSTRACT

A physically based failure criterion distinguishing the different effects of hydrostatic tensile and compressive stress is developed for homogeneous and isotropic materials. It is reasonably assumed that failure is related to the shape and volume change of isotropic materials at the macroscopic scale. Shape distortion and volume dilation (i.e., hydrostatic tension) are capable of producing material failure, while volume contraction (i.e., hydrostatic compression) has an impeding effect upon failure initiation. The different roles of hydrostatic tension and compression may lead to differences of the applied failure functions. In the present study, two failure formulas are proposed, considering the different effects of hydrostatic stress upon yielding/fracture of homogeneous and isotropic materials separately. The good agreement of the present theory with a large number of experimental data is observed, and the present theory shows a wide range of applicability.

1. Introduction

A considerable amount of theoretical and experimental research work on the strength theory of homogeneous and isotropic materials has been done since the beginning of classical mechanics. A brief historical summary of strength and failure treatments was given by Yu (2002). However, so far there has not been a widely acknowledged failure criterion applicable to all types of isotropic materials under general conditions (Christensen, 2016). Much more effort needs to be made in order to develop a physically based general failure theory for homogeneous and isotropic materials.

Isotropic materials can be classified as being either ductile or brittle, whose mechanical behaviors are quite different. If the material is ductile, failure is usually specified by initiation of yielding, whereas for brittle materials it is specified by fracture. Hence the term "failure" is used in a broad sense including both the conditions of yielding and fracture in the present paper.

Among all the macroscopic failure criteria proposed historically, the maximum normal stress/strain, Coulomb-Mohr, Tresca and Mises failure criteria are the most famous and frequently used ones. The maximum normal stress criterion is valid in some stress states for brittle materials, but not suitable for ductile materials. Saint-Venant and Poncelet argued that normal strain should be used instead of normal stress (Timoshenko, 1953), yet the maximum normal strain theory cannot be applied to ductile materials, too. The Coulomb-Mohr criterion describes the failure of many brittle materials. It is a two-property (tensile strength and compressive strength) form and is very popular in engineering practice due to its ease of use. The linear Coulomb-Mohr criterion can be expressed as σ_1/T-$\sigma_3/C = 1$, where σ_1 and σ_3 are the maximum and minimum principal stresses respectively. The disadvantage of this criterion is that it does not take into account the effect of the intermediate principal stress (σ_2) on material failure (Yu, 2004). The Tresca criterion can be regarded as a special case of the Coulomb-Mohr criterion. The latter reduces to the Tresca form as long as the tensile strength equals the compressive strength. The Mises criterion was originally put forward as a better curve-fitting form for ductile materials than the Tresca criterion. Later, it has been found that yielding is caused by distortional states in typical ductile materials, and the Mises form represents a critical value of the distortional energy stored in isotropic materials (Christensen, 2013).

The Mises criterion is the most popular criterion for ductile materials. However, sometimes it is rather difficult to judge whether a specific kind of material is of ductile or brittle nature. The percent elongation and the percent reduction in area are commonly served to specify the ductility of a material. Any material can be subjected to large percent elongation and percent reduction in area before fracturing is called a ductile material, otherwise it is brittle. However, "large" is an ambiguous word which is incapable of quantifying the material type. In fact, there seems to be no clear distinction between ductile and brittle materials. For example, at low temperature materials become more brittle whereas they are more ductile if the temperature rises (Hibbeler, 2014). Therefore, it is appealing to develop a unified failure criterion

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applicable to all types of homogeneous and isotropic materials.

It is known that all stress states of isotropic materials can be built up from the superposition of states of shape distortion and volume change (Christensen, 2013). The distortional part is associated with the second invariant of the deviatoric stress tensor J_2, while the dilatation part is concerned with the first invariant of the stress tensor I_1 or the hydrostatic stress ($\sigma_h = I_1/3$). The reason why the Mises theory is only satisfactory for typical ductile materials is that it merely considers the distortional part related to J_2. Nevertheless, experiments have demonstrated the effect of hydrostatic pressure on the failure behavior of both ductile materials such as aluminum (Spitzig and Richmond, 1984) and brittle materials such as concrete and soil (Yu, 2004). Therefore, it is necessary to include the effect of I_1 in the failure criterion. In other words, both distortion and dilatation influence the failure behavior of homogeneous and isotropic materials.

As the strain energy contains two parts which can both result in failure, namely the distortional energy and the dilatational energy, it seems reasonable to replace the Mises theory with the maximum strain energy theory (Beltrami, 1889). Unfortunately, the theory proves to be impracticable because isotropic materials do not fail under hydrostatic pressure, at least not within the usual range of pressures for engineering materials (Christensen, 2013). Nevertheless, the insight that failure is related to the shape and volume change of isotropic materials does make sense.

Lord Kelvin held the view that "It may be inconceivable that any amount of uniform pressure applied to the surface of a solid sphere of isotropic material would cause it to rupture, but it is also very difficult to believe that a uniform tension, if it could be applied to its surface, would not, were it indefinitely increased, produce rupture" (Thomson and Tait, 1879). In addition, typical test data show that superimposed hydrostatic pressure has a strengthening effect on any other stress state (Christensen, 2013). Therefore, it is reasonable to hypothesize that volume contraction under hydrostatic compressive stress impedes failure initiation while volume expansion under hydrostatic tensile stress promotes material failure. Several stress invariants based I_1-J_2 failure criteria have been formulated for homogeneous and isotropic materials.

The quasi-linear form $\alpha I_1 + \beta\sqrt{J_2}$ was advocated by Drucker and Prager (1952). The coefficients α and β are calibrated by the tensile and compressive strengths T and C, which gives

$$\frac{1}{2}\left(\frac{1}{T} - \frac{1}{C}\right)I_1 + \frac{\sqrt{3}}{2}\left(\frac{1}{T} + \frac{1}{C}\right)\sqrt{J_2} \leq 1. \tag{1}$$

This criterion is capable of considering the different effects of positive and negative hydrostatic stress (i.e., $I_1 > 0$ or $I_1 < 0$) on material failure. Brunig (1999) used an extended version of the Drucker-Prager model to simulate the hydrostatic stress effect on material plasticity. Nevertheless, the criterion gives incorrect results in some particular cases. For instance, it predicts that all materials with $T/C \leq 1/3$ can support unlimited stress magnitudes in equi-biaxial compression, which is certainly impossible (Christensen, 2013).

Silano et al. (1974) proposed a more general form to account for the hydrostatic stress dependency:

$$\sum_{i=0}^{N} \alpha_i (I_1)^i + \beta\sqrt{J_2} = 1. \tag{2}$$

The equation reduces to the Mises and Drucker–Prager form when $N = 0$ and $N = 1$ respectively. Pae (1977) applied Eq. (2) to predict the yield behavior of two different kinds of polymers. The similar form was used by Khan et al. (1991) to predict the failure behavior of Berea sandstone.

The quadratic form $\alpha I_1 + \beta J_2$ was proposed by Stassi-D'Alia (1967) and the criterion is given by

$$\left(\frac{1}{T} - \frac{1}{C}\right)I_1 + \frac{3}{TC}J_2 \leq 1. \tag{3}$$

This criterion also considers the different effects of positive and negative hydrostatic stress. It predicts the pure shear strength $S = 2.24T$ for very brittle materials with $T/C = 1/15$. Nevertheless, experiments have shown that the pure shear stress needed to fracture a specimen during a torsion test is approximately the same as the tensile stress needed in simple tension for very brittle materials (Hibbeler, 2014), i.e., $S \approx T$. Clearly the shear strength is significantly overestimated by the Stassi-D'Alia criterion.

Very recently, an improved theory for ductile materials based on the Mises hypothesis has been developed by Barsanescu and Comanici (2017). The failure criterion is in the form of $\alpha I_1 + \beta I_2$, where I_1 and I_2 are the first and second invariant of the stress tensor. This criterion can be used for ductile materials under different stress states. However, it predicts the hydrostatic compressive strength is identical to the hydrostatic tensile strength, which is not supported by experimental evidence. In fact, the authors themselves admitted that a notable disadvantage of their theory is that it cannot distinguish between hydrostatic tension and hydrostatic compression.

Some researchers suggested including the third invariant of the deviatoric stress, J_3, in the expression of the failure function, forming the I_1–J_2–J_3 theory (Brunig et al., 2000; Hu and Wang, 2005). Bai and Wierzbicki (2008) discussed a pressure and Lode dependent metal plasticity model and its application in failure analysis of aluminum. Gao et al. (2011) described a new stress-state dependent plasticity model for isotropic materials, and presented its finite element implementation for a 5083 aluminum alloy.

A large number of previous studies used a single equation to characterize material failure behavior (e.g., the Drucker-Prager criterion and the Stassi-D'Alia criterion). However, a single mathematical formula calibrated by the tensile and compressive strengths leads to the problem of C/T dependence (Hu and Wang, 2005):

It is known that all stress states can be decomposed into a dilatational part (associated with I_1 or hydrostatic stress σ_h) and a distortional part (associated with J_2). For example, the uniaxial compressive strength C is associated with hydrostatic compression and distortion, while the uniaxial tensile strength T is associated with hydrostatic tension and distortion. Consider the stress state of equi-triaxial tension. Since the uniaxial compressive strength C appears in the failure function, it is predicted that failure under equi-triaxial tensile stresses depends on the compressive strength C. However, this is unacceptable from the physical point of view because under equi-triaxial tension, material failure depends only on hydrostatic tension instead of hydrostatic compression. Similarly, the uniaxial tensile strength T should not appear in the failure function in the case of equi-triaxial compression.

Therefore, in order to distinguish the positive and negative effects of hydrostatic stress as well as to avoid the C/T dependence, it might be appropriate to use different formulas to characterize material failure behavior for hydrostatic tension and compression respectively. In the present study, two different failure functions corresponding to hydrostatic tension and compression are developed. It is shown that the present failure theory is in good agreement with a large number of experimental data and has great flexibility and generality in application.

2. Development of the failure criterion

The stress invariants method applied here is a classical method to define the failure function of homogeneous and isotropic materials (Yu, 2004). The basic three stress invariants are given by

$$I_1 = \sigma_{ii}, \tag{4}$$

$$I_2 = \sigma_{ij}\sigma_{ij}, \tag{5}$$

$$I_3 = |\sigma_{ij}|. \tag{6}$$

The first invariant of the stress tensor I_1 is connected with the