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Adomian Decomposition Method applied to anisotropic thick plates in bending

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ABSTRACT

The aim of this report is to apply the Adomian Decomposition Method (ADM) to anisotropic rectangular moderately thick plates under linear bending using a first-order shear deformation theory. Given the plate's homogeneity, the transverse displacement and rotations degrees-of-freedom are globally interpolated by kinematically admissible polynomials functions. The Rayleigh-Ritz Method is employed to solve the equilibrium equations. As proposed by the original Adomian Decomposition Method, the degrees-of-freedom are expanded into an infinite series, but the differential operator is additively decomposed following constitutive hierarchy. The initial solution refers to a lower lower constitutive symmetry in the operator decomposition and is enhanced by non-isotropic ones. The recursive system requirements in order to have convergence are presented and they depend only on the anisotropic index. The results obtained by the methodology are discussed and compared to those found in the literature. Solutions for different boundary conditions types as well as different thickness under uniform loading are presented so as to provide benchmark solutions. Furthermore, analysis of the anticlastic curvature and the effects of the transverse shear at corners with soft simply-supported edges, largely ignored by the literature, are shown and discussed.

1. Introduction

In order to study the linear bending of anisotropic moderately thick plates, the Mindlin plates theory Mindlin (1951); Reddy (2007) and the Rayleigh-Ritz method Bhat (1985); Liew and Wang (1993) are employed in the present work. An additive constitutive decomposition is applied so as to extract an isotropic part from an anisotropic constitutive tensor and, using the Adomian Decomposition Method Adomian (1994) (ADM), isotropic solutions are recursively enhanced by their anisotropic residuals.

The motivation of this study regards the increasing use of anisotropic materials on structural elements. In spite of anisotropic plate bending being a well-established research topic, the solution methodology proposed herein has potential to be applied to new classes of problems, linear or non linear, as it will be shown. The present version of ADM has already been successfully applied to anisotropic Lisbôa et al. (2017) and laminated Lisbôa and Marczak (2017) thin plates. This new study addresses the ADM as a solution methodology to obtain the mechanical response of anisotropic moderately thick plates.

The Mindlin's theory is a well-known first-order shear deformable theory (FSDT) in the literature. Given the resulting discordance between the theoretical shear stress (parabolic) and the one obtained by FSDTs (constant), the transverse shear must be weighted by a correction factor Reddy (2007); Jemielita (2002). There are also other theories that consider the transverse shear strain in orders higher than Mindlin's one, which avoid the need of shear correction factors. Nevertheless, boundary conditions may lack physical explanation and should be cautiously analysed Hanna and Leissa (1994); Lo et al. (1977). In the FSDT, the boundary conditions have direct physical meaning and do not have the condensation of the transverse shear and twisting moment in the boundary conditions as in CPT. In the engineering context, therefore, FSDT still is the most useful among the usual plate theories.

The mechanical behaviour of anisotropic thick plates has been studies by several researchers. One may cite the work of Srinivas and Rao (Srinivas and Rao (1970)), which presents solutions for thick orthotropic rectangular plates for bending, vibration and buckling. Wu and Wardenier (Wu and Wardenier (1998)) have also presented a deep investigation on this subject. Umasree e Bhaskar (Umasree and Bhaskar (2005)) have introduced a new methodology to obtain analytic solution of anisotropic and thick plates. Carrera Carrera (2002) has discussed the state-of-the-art in anisotropic laminated plates and shells. The author has also presented, as a sequel of the previous paper, a unified

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compact formulation for plates, known as Carrera's Unified Formulation (CUF) Carrera (2003). Based on nuclei, such formulation can virtually develop all known plate theory and also describes the types and classes of plate theories. The application of CUF is concentrated in sandwich, laminated and multilayered plates and shells Ferreira et al. (2011, 2014); Tornabene et al. (2016).

In most stress-strain analysis, the generalized Hooke's law is employed to obtain the stresses in the domain. In plates, however, a fully triclinic material is not achievable due to the constitutive symmetry plane parallel to the plate's mid-surface Alternbach (1998). Essentially, a constitutive symmetry is defined by the number and position of the symmetric planes while the concept of constitutive hierarchy regards relationships among the symmetry sets. If a symmetry set is a subgroup of another one, a hierarchy relation is then established and, given the finite number of constitutive symmetries Cowin and Mehrabadi (1995); Forte and Vianello (1996); Chadwick et al. (2001); Ting (2003), a hierarchic tree can be assembled. Based on these symmetries relations, a complementary decomposition which extracts a haigher symmetry from lower one can be derived. Tu Tu (1968) and Browayes & Chevrot Browaeys and Chevrot (2004) have developed two similar decompositions for three-dimensional constitutive tensors using orthogonal basis and projections for each symmetry, respectively. Moreover, they have applied the concept of crystal family instead of crystalline system, used herein and in Tu Tu (1968). Following a procedure similar to the ref. Browaeys and Chevrot (2004), a new set of projections are developed here, because the constitutive tensor used in the FSDT is reduced from the three-dimensional one. Later, these projections are applied to the plate reduced constitutive tensor in order to set up the proposed recursive methodology.

The *pb-2* Rayleigh-Ritz Method (RRM) Bhat (1985); Liew and Wang (1993) is a variation of the usual RRM and it is used in the present work to solve the equilibrium equations. The *pb-2* modification simplifies the imposition of more generic boundary condition set. By using this method, Kitipornchai et al. Kitipornchai et al. (1994) have studied the free vibration behaviour of isotropic thick trapezoidal plates with several boundary conditions sets while Singh and Elaghabash Singh and Elaghabash (2003) have analysed finite displacement of isotropic plates by considering the von Kármán hypothesis. Bhat Bhat (1985) and Liew & Wang Liew and Wang (1993) have utilised the method in order to obtain global static solutions for thin isotropic plates. Liew Liew (1992) has employed the method so as to derive solutions for general problems in thin plates.

The Adomian Decomposition Method (ADM) Adomian (1994) can be described by three main characteristics: (a) the decomposition of the matrix/differential operator into two specific terms - linear and remainder; (b) the expansion of the problem's solution in an infinite series; and (c) the determination of each solution's term in a recursive manner. Abbasbandy Abbasbandy (2003) has improved the Newton-Raphson method applied to non-linear equations by using a modified version of the ADM while Biazar el al. Biazar et al. (2004) and Babolian & Biazar Babolian and Biazar (2002) have demonstrated that the decomposition can be applied to ordinary non-linear equations. Moreover, Abbasbandy Abbasbandy (2006b, a, c), Öziş & Yiklirim Ozis and Yildirim (2008) and Li, (2009) have proved that the methodology is a special case of the Homotopy Perturbation Methods.

The objective of this paper is to analyse the potential of ADM to solve anisotropic thick plates in bending. The methodology was previously applied for anisotropic Lisbôa et al. (2017) and laminated Lisbôa and Marczak (2017) thin plates and here it is further developed to include thick plates. Firstly, the governing equations of the FSDT used are presented. In order to provide an additive decomposition ruled by a constitutive hierarchy, a crystal-symmetry hierarchic tree is assembled and its projections developed. The matrix operator resultant from the application of the RRM is then decomposed and the solution's vector is expanded in an infinite series. A system is then assembled so as to determine the full anisotropic response of the plate where its first step is the solution of an isotropic case, whose properties were extracted from the original constitutive tensor. In the subsequent steps, the anisotropic influence is recursively added back. Given the recursive nature of the method, the requirements for the absolute convergence are also studied. Numeric results are provided, including an analysis on the anticlastic curvature as well as on the transverse shear at corners of soft simply-supported edges.

There exist two main reasons for using ADM in this context: (a) the possibility of obtaining analytic solutions for complex problems, and (b) the separation of the isotropic and anisotropic solutions of the problem. The first point is supported by several papers (Ghosh et al. (2007); Duan and Rach (2011); Al-Hayani (2011); Duan et al. (2013)). These works have dealt with complex linear and nonlinear problems and, by using ADM along with some modifications, the authors have obtained their analytical solution. An fairly comprehensive review on the method's variations as well as its many applications can found in ref. Duan et al. (2012). The second point was considered by Lisbôa and Marczak (2017) for thin plates and is expanded in this paper for thick plates using a FSDT. In addressing this subject, the methodology presented in the present work is applied directly to a matrix differential operator. Moreover, as in Mindlin plate theory the stress state is different from thin plates, thus the constitutive hierarchy is redefined so as to support the constitutive decomposition: a first step of ADM and what ultimately separates the solution into its isotropic and anisotropic parts.

This report is organized as follows. In section two the governing equation of anisotropic moderately thick plates are presented. Section three defines the constitutive hierarchy and their projections, as used in the additive constitutive decomposition. In section four the *pb-2* RRM is outlined and applied to the governing equations. The ADM is introduced and applied to the linear system generated by RRM, in the section five where the convergence criteria of the methodology are also presented. In section six, the solutions obtained are discussed and compared to those found in the literature along with tables containing numeric results for several cases geometry and boundary conditions. The analysis of the anticlastic curvature and shear distribution for soft simply-supported edges are also developed in section six. In the Appendix, a collection of all projections and reduced constitutive tensors for the symmetries considered by the methodology is shown.

Both matrix and index notation are used. Bold-face lower and upper-case denote 1st and 2nd order tensors, respectively, while non-indexed symbols describe scalars. All vectors are defined as column-vector. In index notation, lower and upper-case indices ranges from 1 to 3 and 1 to N, respectively, while Greek indices varies from 1 to 2.

2. Governing equations

Let a homogeneous plate with uniform thickness, h, in a rectangular domain. Its displacement field as described by FSDT is written as Reddy (2007) (Fig. 1)

$$\begin{aligned} U_1(x_1, x_2, x_3) &\doteq -x_3\theta_1(x_1, x_2)\,, \\ U_2(x_1, x_2, x_3) &\doteq -x_3\theta_2(x_1, x_2)\,, \\ U_3(x_1, x_2) &\doteq u_3(x_1, x_2)\,, \end{aligned} \tag{1}$$

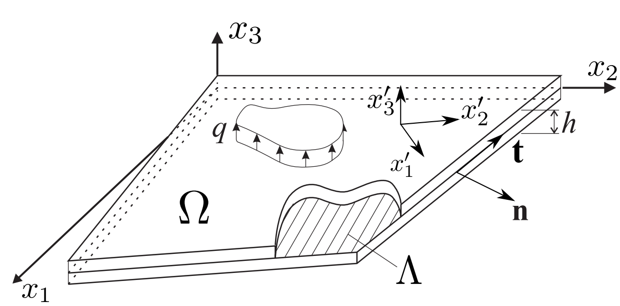

Fig. 1. Plate's domain.