European Journal of Mechanics / A Solids 69 (2018) 231–237

Contents lists available at ScienceDirect

European Journal of Mechanics / A Solids

journal homepage: www.elsevier.com/locate/ejmsol

Dynamics of a beam with both axial moving and spinning motion: An example of bi-gyroscopic continua

Xiao-Dong Yang[a,*], Ji-Hou Yang[a], Ying-Jing Qian[a], Wei Zhang[a], Roderick V.N. Melnik[b]

[a] *Beijing Key Laboratory of Nonlinear Vibrations and Strength of Mechanical Engineering, College of Mechanical Engineering, Beijing University of Technology, Beijing 100124, China*

[b] *The MS2Discovery Interdisciplinary Research Institute, M2NeT Laboratory, Wilfrid Laurier University, 75 University Avenue West, Waterloo, ON, N2L 3C5, Canada*

ARTICLE INFO

Keywords:
Bi-gyroscopic system
Axially moving and spinning material
Bifurcation

ABSTRACT

Classical gyroscopic continua include axially moving materials and spinning structures. The gyroscopic effect of the axially moving material is pronounced via the gyroscopic coupling among the basis functions in the same motional direction. On the other hand, the gyroscopic coupling of the spinning structure acts in the two different directions of motion. In this paper, we study the dynamics of a beam with both axial moving motion and spinning motion as a prototype of bi-gyroscopic continua. The influence of bi-gyroscopic effects on the natural frequencies, modes, and stability is investigated by an analytical method applied to the discretized equations of the axially moving and spinning beam. Distinct bifurcation series of the eigenvalues and corresponding physical interpretations are discussed by numerical display of the modal motions. The complex modes describing both whirling motions and traveling waves are investigated in detail for such bi-gyroscopic system. New interesting phenomena have been analyzed numerically and important conclusions have been drawn for such bi-gyroscopic system.

1. Introduction

From a broader perspective, mechanical structures that can vibrate about a state of mean rotation are classified as gyroscopic dynamic systems. The gyroscopic effect comes from the Coriolis force measured on the rotating frame. Spinning flexible structure is a straightforward example of the gyroscopic continua. Another example is the axially moving material, for which the rotating frame is observed on the slope variation of the overall contour.

In the case of spinning structure as presented in Fig. 1(a), the spinning velocity is $\overline{\Omega}$ and the transverse displacement in Z direction is W, and the corresponding velocity is $\partial W/\partial T$. The Coriolis force on a small element caused by the Z directional motion is $2\overline{\Omega}(\partial W/\partial T)$, along the Y direction. On the other hand, if the displacement and velocity in Y direction is measured, the Coriolis force in Z direction will be generated. Hence, the Coriolis force of the spinning structure makes every element of the continuum vibrating in the YZ plane, showing the elliptic whirling motions. The gyroscopic coupling between the two transverse directions is the feature of the spinning bodies.

In the case of axially moving structure as presented in Fig. 1(b), the axially moving velocity is U and the transverse displacement in Z direction is W. An arbitrary element of the flexible structure, excluding the supporting ends, involves a rotating velocity $\partial^2 W/\partial X\partial T$. The Coriolis force is then $2U(\partial^2 W/\partial X\partial T)$ along the transverse Z direction. Similarly, the motion in the Y direction will cause Coriolis force in the same direction. Hence, contrary to the spinning structures, the two transverse directional motions are not coupled gyroscopically through the Coriolis force for the axial moving structures. Actually, the gyroscopic coupling acts on different basis functions along the same transverse direction.

From the above discussions, two types of gyroscopic continua emerge: spinning structures and axially moving structures. The gyroscopic effects present different phenomena in the two types of gyroscopic continua: (a) the two transverse directions of the spinning structure are gyroscopically coupled by the Coriolis force, which leads to whirling motions in the YZ plane, and (b) the basis functions on the same transverse direction of the axially moving material are gyroscopically coupled by the Coriolis force, which leads to travelling waves in XZ and XY plane, independently.

The presence of the gyroscopic terms in the governing equations of the gyroscopic continua limits analytical results, but enriches the dynamical behaviors dramatically. In the study of spinning well balanced axisymmetric structures, like spinning disks (Fang et al., 2014; Genta, 2005), spinning rings (Cooley and Parker, 2014; Genta and Silvagni,

* Corresponding author.
E-mail address: jxdyang@163.com (X.-D. Yang).

https://doi.org/10.1016/j.euromechsol.2018.01.006
Received 21 May 2017; Received in revised form 19 January 2018; Accepted 22 January 2018
0997-7538/

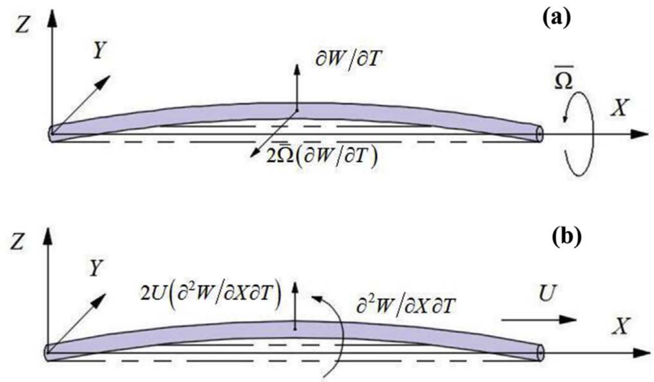

Fig. 1. The two types of gyroscopic continua. (a) Spinning structure; (b) Axially moving structure.

2013; Kim and Chung, 2002), spinning beam (Wu et al., 2014) and spinning disk-spindle systems (Parker and Sathe, 1999a, b), it is found that the natural frequency of each mode for the stationary structure branches into two because the spinning motion couples the two transverse directions and transfers the two isolated equal frequencies into distinct ones. In general, these branches represent the natural frequencies for the forward and backward whirling motions. In the study of axially moving materials, like pipes conveying fluid (Païdoussis, 1998; Yu et al., 2014), axially moving strings (Chen, 2005; Parker, 1998; Wickert and Mote, 1989) and axially moving beams (Öz and Pakdemirli, 1999; Parker, 1998), it is concluded that the natural frequencies will decrease with the increasing axial velocity until the divergence occurs, beyond which the system loses its stability. However, due to the gyroscopic terms, the axially moving structure may regain stability and, with further increase of velocity, loses stability by flutter. Due to the gyroscopic effect, the 'travelling waves' of the axially moving material during modal motions have been studied, which are different from the 'standing waves' of the static structure (Yang et al., 2016). From the Galerkin discretized point of view, the 'standing waves' are maintained because the basis functions (sine functions for the both end supported case) are exactly the solutions of the static structure and 'travelling waves' are caused due to the fact that the solutions are composed of coupled basis functions with phase differences.

The differences of the spinning structure and axially moving material as mono-gyroscopic continua have been listed in Table 1. The gyroscopic dynamics found in the two types of gyroscopic continua are different for both the coupling style and the vibration phenomena. One question then arises from the comparison: what dynamics will arise if the structure undergoing both spinning and axial motion? In the present study, we answer this question by proposing a novel idea of the bi-gyroscopic effect: a gyroscopic effect from both spinning and axially moving motion, which belong to two different types of gyroscopic couplings.

In the engineering field, the structures with both spinning and axial motion are used, among other applications, as a component in drilling machines (Arvajeh and Ismail, 2006; Rincon and Ulsoy, 1995) and drilling oil pipes (Pei et al., 2013; Zhang and Miska, 2005). In the

Table 1
Comparison of the two types of gyroscopic continua.

	Modal motions	Coupling	Gyroscopic features
Spinning structure	Whirling	Two transverse directions coupled	One frequency braches into two.
Axially moving structure	Travelling wave	Basis functions in the same direction coupled	Two stable regions exist; Bifurcate from stability to divergence, and then from restability to flutter.

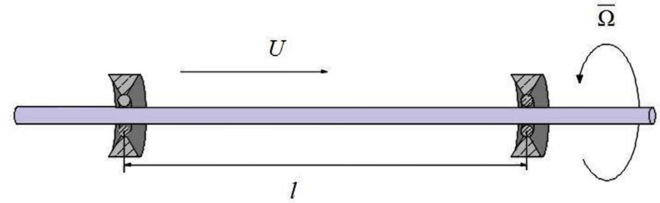

Fig. 2. Diagram of beam undergoing both spinning and axial moving motion.

available dynamical studies of the drilling slender continua, the axial motion has been usually neglected. On the other hand, from the theoretical point of view, a combination of spinning and axial motion may lead to rich dynamics of the gyroscopic system. In this paper, a prototype of bi-gyroscopic continua is investigated based on the beam model undergoing both spinning and axial motion. The features of the bi-gyroscopic couplings and modal motions are studied and discussed. The bifurcation series of the dynamics based on the eigenproblem is analyzed in detail, which may provide a foundation for further investigations on structures with bi-gyroscopic couplings.

2. System model

A circular flexible beam simply supported by two joints with distance l, is undergoing both spinning motion with constant velocity $\overline{\Omega}$ and axial moving motion with constant speed U, as shown in Fig. 2. The stiffness, cross section area and density of beam material is EI, A and ρ, respectively. To derive the displacements of the flexible beam, two sets of rotating reference coordinates are used: $OXYZ$ is a spinning reference frame with spinning velocity $\overline{\Omega}$ along X axis; $PX_1Y_1Z_1$ is reference frame with both spinning and axial moving motion. Without the spinning velocity, the two reference frames recover the Euler and Lagrange descriptions of the axially moving material, respectively. The deflections of an arbitrary moving beam element can be observed in the $PX_1Y_1Z_1$ frame. However, the final visual displacements of the axis line of the beam are measured in the rotating frame $OXYZ$ (see Fig. 3).

The transformation relations of the two rotating references are

$$X_1 = X + UT, \quad Y_1 = Y, \quad Z_1 = Z. \tag{1}$$

If the transverse displacements V and W of an arbitrary point on the beam are defined on Y_1 and Z_1 direction, respectively, the position vector of such point P_1 in the $OXYZ$ frame is

$$\begin{aligned}\mathbf{OP}_1 &= (UT)\mathbf{i} + \mathbf{PP}_1 = (UT)\mathbf{i} + V(X_1, T)\mathbf{j} + W(X_1, T)\mathbf{k} \\ &= (UT)\mathbf{i} + V(X + UT, T)\mathbf{j} + W(X + UT, T)\mathbf{k}.\end{aligned} \tag{2}$$

Further, the velocity can be derived by the first derivative of (2) as

$$\mathbf{v} = U\mathbf{i} + \left(\frac{\partial V}{\partial T} + U\frac{\partial V}{\partial X} - \overline{\Omega}W\right)\mathbf{j} + \left(\frac{\partial W}{\partial T} + U\frac{\partial W}{\partial X} + \overline{\Omega}V\right)\mathbf{k}. \tag{3}$$

Then the kinetic energy and potential energy are, respectively,

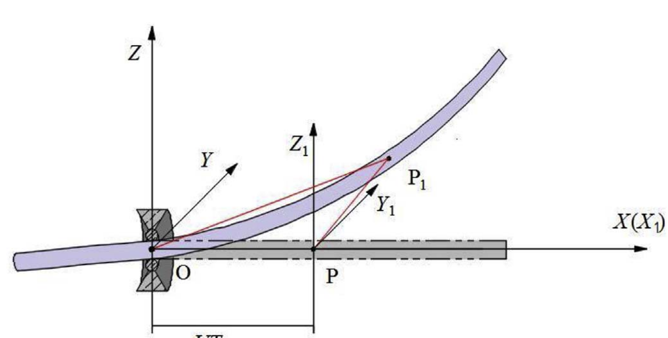

Fig. 3. Deformation of the structure.