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Finite element dynamic analysis of beams on nonlinear elastic foundations under a moving oscillator

C. Rodrigues[a], F.M.F. Simões[b,*], A. Pinto da Costa[b], D. Froio[c], E. Rizzi[c]

[a] Instituto Superior Técnico, Universidade de Lisboa, Avenida Rovisco Pais 1, 1049-001 Lisboa, Portugal
[b] CERIS, Instituto Superior Técnico, Universidade de Lisboa, Avenida Rovisco Pais 1, 1049-001 Lisboa, Portugal
[c] Department of Engineering and Applied Sciences, University of Bergamo, Viale G. Marconi 5, 24044 Dalmine (BG), Italy

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ABSTRACT

This paper presents a study on the dynamic response of beams on elastic foundations, subjected to a uniformly moving oscillator. Using a finite element model programmed within a MATLAB environment the response of the system is studied for three different types of mechanical behaviour of the foundation: (a) linear elastic (classical Winkler model), (b) nonlinear elastic (in which the foundation reaction displays a cubic dependence on the beam displacement) and (c) bilinear elastic (with different compressive and tensile stiffnesses). The effects of the oscillator's natural frequency and velocity and of the foundation's stiffness and damping are investigated. In particular, critical velocities of the oscillator and ranges of velocities for which the system is dynamically unstable are numerically determined for the first time in the above mentioned nonlinear cases.

1. Introduction

The interaction between elastic structures and moving mechanical systems has been a topic of interest for well over a century and, nowadays, such interest has been even more stimulated by the current progresses in transportation systems. For some types of soils, modern high-speed trains are able to move with velocities comparable with the minimal phase velocity of wave propagation in the elastic supporting structure (Metrikine and Verichev, 2001), causing vibrations whose amplitudes may be significantly higher than the deflections due to static loads. These vibrations may damage the supporting structures and seriously influence the comfort and safety of the passengers. Similarly, high vibrational amplitudes may be also encountered in other problems of mechanical or structural engineering, such as high-speed precision machining, advanced propulsion concepts like railguns, aircraft carriers or transportation cables (Yang et al., 2000). Therefore, it is interesting to study the dynamic response of structures supporting moving mechanical systems in order to mitigate the above mentioned effects.

The elastic structures are most commonly represented by a finite or infinite beam supported by a uniform or non-uniform viscoelastic linear or nonlinear foundation. Various foundation models such as Winkler, Pasternak, Vlasov or Reissner and either Euler-Bernoulli or Timoshenko beam models have been used. Concerning the moving system, three types of models have been mainly employed in the literature, thus defining different mechanical problems: (i) the *moving oscillator* (spring-mass-dashpot) problem, which is considered when the stiffness of the moving subsystem is finite and its inertial effects are not negligible; (ii) the *moving mass* problem, which may be conceived as a subcase of the moving oscillator problem when the stiffness of the moving subsystem approaches infinity; (iii) the *moving load* problem, which also neglects the inertia of the moving subsystem.

The nonlinearity of the moving mass and moving oscillator problems poses some mathematical difficulties that do not occur in the case of the moving load problem. Many examples on how these mathematical difficulties have been tackled may be found in the literature related to the *moving mass* problem and some of them are briefly reported next. Hutton and Counts (1974) used sine series for the spatial approximation of the beam deflection and numerically solved the resulting time dependent system of second-order ordinary differential equations. Stanišić (1985) presented an approximated method based on an eigenfunction series expansion for the beam deflection with time dependent coefficients. An alternative formulation was followed in the work by Sadiku and Leipholz (1987), who proposed a Green's function approach for the moving load and the moving mass problems for a finite beam, leading to an integro-differential equation in terms of the beam displacements whose solution can be obtained to any desired degree of accuracy. Lee (1996) investigated the onset of the separation between the moving mass and the beam by using the modal analysis method and solving the set of coupled ordinary differential equations through the fifth-order Runge-Kutta scheme. Analytical solutions for

* Corresponding author.
E-mail address: fernando.simoes@tecnico.ulisboa.pt (F.M.F. Simões).

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the case of an infinite beam excited either by a moving harmonic force or by a moving mass, were provided by Duffy (1990) using Fourier and Laplace transforms. The stability of the oscillations of a mass moving at constant velocity along an infinite Euler-Bernoulli beam was studied by Denisov et al. (1985) and by Metrikine and Dieterman (1997). In both works it was proved that besides resonance, occurring when the velocity of the moving mass becomes equal to the minimum phase velocity of waves in the beam-foundation system, also dynamic instabilities may occur induced by anomalous Doppler waves radiated by the moving object. These instabilities are characterized by vibrations of the system whose amplitude grows exponentially in time. The region of instability of the system was determined with the help of the D-decomposition method. The same technique was applied by Wolfert et al. (1998) in the study of the stability of two masses moving at a constant distance along an Euler-Bernoulli beam on a viscoelastic foundation.

The *moving oscillator* problem has also been the subject of many works. Pesterev and Bergman (1997a, b) considered a linear conservative finite beam carrying a moving undamped oscillator and proposed a mathematical formulation that allowed for the solution of the interaction problem in a series of the eigenfunctions of the elastic system. Then, the time-dependent coefficients of the expansion were obtained by solving a set of linear ordinary differential equations. A later extension of the method incorporated proportional damping (Pesterev and Bergman, 1998). Based on the previous approach, Omenzetter and Fujino (2001) examined the vibrations of a proportionally damped linear moving multi-degree-of-freedom oscillator interacting with the beam at several contact points. Similarly to the approach of Sadiku and Leipholz (1987), Yang et al. (2000) analysed a spring-mass moving oscillator, solving by numerical integration the final integral equation for the beam displacement. Fourier transforms, for both space and time variables, were used by Bitzenbauer and Dinkel (2002) to find the dynamic response of a linear multi-degree-of-freedom system moving along an infinite beam; the system was excited by the vertical imperfections of the track and its initial conditions were neglected. Muscolino and Palmeri (2007) studied the response of beams resting on viscoelastic foundations and subjected to moving oscillators. Metrikine and Verichev (2001) investigated the stability of an oscillator moving at constant velocity along an infinite Timoshenko beam on a foundation and determined the instability domains in the space of the system parameters by employing again the D-decomposition method. Later they also studied the stability of a moving bogie (Verichev and Metrikine, 2002). Galerkin's method has also been applied to reduce the partial differential equations of motion to a set of coupled ordinary differential equations containing periodic coefficients that is numerically solved. This approach was followed by Yoshimura et al. (1986) for a simply supported beam subjected to a moving oscillator including the effects of geometric nonlinearity, by Katz et al. (1987) for a simply supported beam subjected to a moving load whose amplitude is deflection dependent, and by Ding et al. (2014) in the study of the dynamic response of the oscillator-pavement coupled system by modelling the pavement as a Timoshenko beam resting on a six-parameter nonlinear foundation. The Finite Element Method (FEM) was also used to obtain the response of beams resting on elastic foundations and subjected to moving oscillators. Hino et al. (1985) studied the vibration of finite nonlinear beams subjected to a moving oscillator by using the FEM and Newmark's implicit time integration algorithm. Lin and Trethewey (1990) also presented a FEM formulation for the dynamic analysis of elastic beams subjected to a one-foot and a two-foot spring-mass-damper moving systems. A FEM approach was also employed by Chang and Liu (1996), who analysed the vibration of a nonlinear beam on elastic foundation subjected to an oscillator moving on a randomly varying in space beam profile.

The problem of a moving load with harmonically varying amplitude has also been studied by several authors; among those, noteworthy to be mentioned are the works by Mathews (1958), Chonan (1978), Bogacz et al. (1989), Chen et al. (2001), Chen and Huang (2003) and, more recently, Froio et al. (2017). Some analytical and numerical solutions to other types of dynamical problems that involve harmonic loading, but not moving along the foundation, have also been proposed in the literature (e.g. Abu-Hilal (2003), Coşkun (2003), Younesian et al. (2012) and Saadatnia et al. (2017)).

In the present paper the finite element method is used in the study of the transverse transient response of a simply supported Euler-Bernoulli beam resting on a viscoelastic foundation and interacting with a moving oscillator. The oscillator moves at constant velocity along the longitudinal beam axis. The response of the system is studied for three types of mechanical behaviour of the foundation: (a) *linear elastic* (classical Winkler model) in Section 3, (b) *nonlinear elastic* (in which the foundation reaction displays a cubic dependence on the beam displacement) in Section 4 and (c) *bilinear elastic* (with different compressive and tensile stiffnesses) in Section 5. The use of the finite element method is a convenient and more practical alternative to the analytical methods used in many of the works mentioned above. It also shows the advantage of solving nonlinear problems for which analytical solutions are not available. The effects of the oscillator's natural frequency and velocity and of the foundation's stiffness and damping are investigated. In particular, critical velocities of the oscillator and ranges of velocities for which the system turns out dynamically unstable are numerically determined for the first time in the above mentioned nonlinear cases. Whenever possible, the obtained results are validated by comparison with previous outcomes from the literature. The present study may be easily extended to the case of multiple moving oscillators.

2. Finite element method formulation

Consider an Euler-Bernoulli beam finite element, of uniform height h and length l, on a viscoelastic foundation defined between two generic nodes i and j, as shown in Fig. 1. The vector of the (four) generalized displacements of the finite element is

$$\mathbf{q}^e = \{q_1 \quad q_2 \quad q_3 \quad q_4\}^{\mathrm{T}} \tag{1}$$

and the transverse displacement field $w(x, t)$ in each finite element is defined by

$$w(x, t) = \{N_1(x) \quad N_2(x) \quad N_3(x) \quad N_4(x)\}\begin{Bmatrix} q_1(t) \\ q_2(t) \\ q_3(t) \\ q_4(t) \end{Bmatrix} = \mathbf{N}^e(x)\,\mathbf{q}^e(t), \tag{2}$$

where the shape functions $N_i(x)(i = 1,\ldots,4)$ are defined by

$$\begin{aligned} N_1(x) &= 1 - 3\left(\tfrac{x}{l}\right)^2 + 2\left(\tfrac{x}{l}\right)^3 \\ N_2(x) &= x\left(1 - \tfrac{x}{l}\right)^2 \\ N_3(x) &= 3\left(\tfrac{x}{l}\right)^2 - 2\left(\tfrac{x}{l}\right)^3 \\ N_4(x) &= x\left(\left(\tfrac{x}{l}\right)^2 - \tfrac{x}{l}\right). \end{aligned} \tag{3}$$

A single-degree-of-freedom oscillator with mass m, stiffness k and

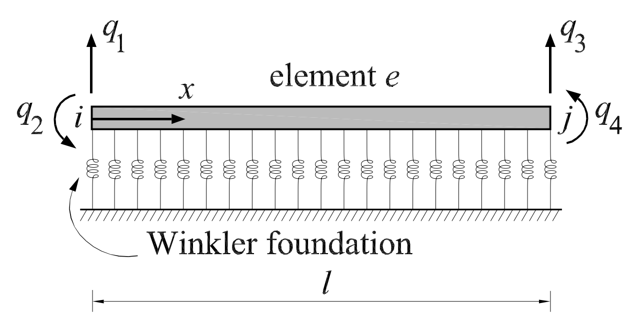

Fig. 1. Euler-Bernoulli beam finite element on an elastic foundation.