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Propagation of a Dugdale crack between two orthotropic half-planes

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ABSTRACT

The aim of this work is to study the propagation of a Dugdale crack between two orthotropic half-planes. The two mediums are of the same unidirectional composite material. The fibers are oriented symmetrically with respect to the interface. The applied propagation criteria are deduced from the revisited Griffith theory (Ferdjani, H. and J.-J. Marigo, European Journal of Mechanics - A/Solids, 2015. **53**: p. 1–9). To resolve the problem, the Lekhnitskii-Eshelby-Stroh representation is used (Suo, Z. Proc. R. Soc. Lond. A 427, 331–358 (1990)). The fracture threshold and the evolution of the applied load are determined for different cases of fiber orientation and crack length. A comparison with the Griffith's model is also presented.

1. Introduction

The problem of interfacial cracks between anisotropic materials has been studied by many authors. In his work, Gotoh (1967) discussed some problems of bonded dissimilar anisotropic plates with cracks along the bond. The elastic-plastic solution, with the Dugdale's assumption of confined plastic deformation along the crack, is also given. The problem of a flat crack of infinite length and constant finite width between bonded dissimilar anisotropic materials has been examined by Clements (1971). The stress distribution throughout the materials has been determined when the crack is subjected to a non-uniform applied stress. Qu and Bassani, (1989) considered the problem of an interface crack between two anisotropic elastic solids. A necessary and sufficient condition for the absence of oscillations in the singular crack-tip fields has been derived. A Griffith crack lying along the interface between anisotropic elastic solids has been analyzed by Bassani and Qu, (1989). Explicit solutions for the displacement and stress fields have been obtained when the crack-tip fields are non-oscillatory. Suo (1990), gives a detailed collection about the mathematical approach of studying the interfacial crack between anisotropic materials, treating oscillatory and non-oscillatory singularity. From all these studies, only Gotoh addressed the problem of a Dugdale crack.

This paper investigates the bi-dimensional problem of the propagation of a Dugdale interfacial crack between two orthotropic half-planes. The two mediums are of the same unidirectional composite material. The fibers are oriented symmetrically with respect to the interface. The loading is such that the crack is in mode I. This particular problem was chosen as an example of mode I crack propagation problem in anisotropic material. There are two main differences between this work and Gotoh's paper:

- First, Gotoh used the Dugdale's assumption of a narrow plastic band ahead of the crack tip (Dugdale, 1960). Whereas in this work, the Dugdale's model is a particular case of the cohesive zone model or the Barenblatt's model (Barenblatt, 1962) for brittle materials. In other words, the Dugdale's model consists in constant cohesive forces acting on the crack faces with no plastic yielding.
- Second, Gotoh has not studied the crack propagation problem, while it is considered in this paper.

The propagation of a Dugdale crack has already been studied in several works. For the mode I case, Ferdjani et al. (2007) studied a crack in an infinite isotropic medium under uniform traction. For the mode III case, Ferdjani and Marigo (2015) and Ferdjani (2009, 2013) considered a crack in a semi-infinite isotropic medium, in an infinite isotropic strip, and at the interface of a strip and a half-plane constituted of different isotropic materials under uniform anti-plane shearing respectively. For the mixed mode case, Ferdjani and Marigo (2015) studied a crack at the interface of a strip and a half-plane constituted of the same isotropic material under uniform traction.

For a crack along an interface between dissimilar anisotropic media, the crack-tip singularity is generally of oscillatory type. For particular types of bi-material, Qu and Bassani, (1989) have shown that the crack tip is free of oscillation. It is shown in the following that the considered problem is one of those special cases.

This study is performed in the framework of the revisited Griffith

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theory (Francfort and Marigo, 1998) for brittle fracture. The Dugdale's model and the crack propagation criteria are established using an energy minimization principle with a surface energy density of Barenblatt type (Ferdjani and Marigo, 2015). The Lekhnitskii-Eshelby-Stroh representation is used to solve the problem (Suo 1990).

The paper is organized as follows. The Dugdale's model is presented in section 2. In section 3, the studied structure is depicted and the crack propagation criteria presented. In section 4, the resolution method is exposed. Section 5 is devoted to numerical results consisting in a parametric study of the problem and a comparison with the Griffith's model.

2. The Dugdale's model in the mode I case

For a crack in a homogeneous medium, the general cohesive zone model has been established by Ferdjani and Marigo, 2015 in the mixed-mode case. Following the same procedure, it can be derived for an interfacial crack in the mode I case. The main characteristics of the model are given below. Details of the method are exposed in Ferdjani and Marigo, (2015).

Consider the plane elasticity problem of a body constituted of two parts bonded through an interface Γ across which the displacement can be discontinuous. The two parts are made of different materials with a linear elastic behavior. The imposed loading causes the propagation of a crack along the interface. Let u_n be the jump of the normal displacement at a point of the crack path, called *gap*. Also, let σ be the normal stress vector on Γ, called *cohesive force*.

The surface energy density φ is defined on $[0, +\infty[$ by Ferdjani et al. (2007):

$$\varphi(u_n) = \begin{cases} \frac{G_c u_n}{\delta_c}, & if \quad u_n \le \delta_c \\ G_c, & if \quad u_n \ge \delta_c \end{cases} \tag{1}$$

In equation (1), G_c is the critical energy release rate of the Griffith theory, whereas δ_c is the characteristic length of the Dugdale's model. The ratio G_c/δ_c has the dimension of a stress, called *critical stress* σ_c:

$$\sigma_c = \frac{G_c}{\delta_c} \tag{2}$$

The stress debonding criterion for the onset of the crack is given by:

$$\sigma = \sigma_c \quad on \quad \Gamma_b,$$

where Γ_b is the bonded part of Γ.

In the debonded part Γ_d, the cohesive force σ is equal to σ_c as long as $u_n \le \delta_c$ and vanishes as soon as $u_n \ge \delta_c$. Therefore, Γ_d is divided into two zones: the so-called *cohesive zone* Γ_c in which the cohesive forces are equal to σ_c and the so-called *non-cohesive zone* Γ_0 in which there are no cohesive forces.

3. The studied structure and crack propagation criteria

3.1. The studied structure

Consider an infinite medium composed of two half-planes constituted from the same unidirectional orthotropic composite. The fibers of the upper (lower) half-plane make an angle of α $(-\alpha)$ with the direction of the interface. An initial crack $D = [-a_0, a_0] \times \{0\}$ is on the interface, (Fig. 1). The crack faces are submitted to constant pressure P_0 increasing from zero, and the body forces are neglected. The state of plane stress is assumed.

3.2. Crack onset and propagation

Since the critical stresses of the mediums are higher than that of the interface, assume the crack propagates horizontally along the interface ($\Gamma = (-\infty, -a_0] \times \{0\} \cup [a_0, +\infty) \times \{0\}$). Moreover, for reasons of symmetry, assume the crack propagates along the axis $y = 0$ in a symmetrical manner from the points $(\pm a_0, 0)$. The debonded part Γ_d is the created crack and $x = \pm c$ the position of its tips:

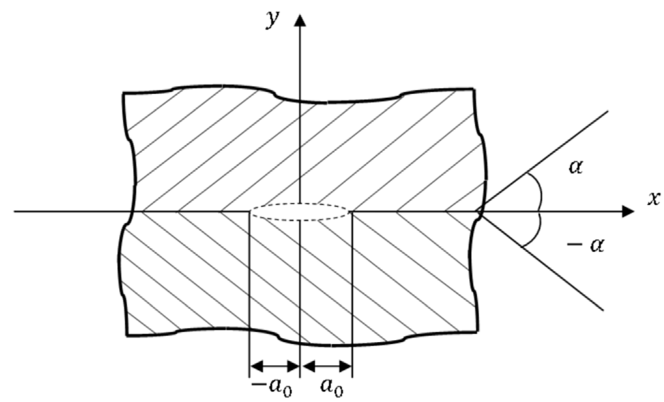

Fig. 1. Geometry of the structure with the initial crack.

$$\Gamma_d = [-c, -a_0] \times \{0\} \cup [+a_0, +c] \times \{0\}.$$

It has previously been seen (paragraph 2) that Γ_d can be divided into two parts:

- The first, close to the crack tip, and named the *cohesive zone*, is subjected to the constant normal cohesive forces σ_c.
- The second, named the *non-cohesive zone*, is close to the initial crack without cohesive force.

These two zones are separated by the limit points $x = \pm a$. Noting that the values of c and a depend on the value of the loading P_0 with assumption $c \ge a \ge a_0$. At the beginning of loading, the initial conditions are: $c = a = a_0$ (Fig. 2).

In the present case, the crack growth follows two phases: the cohesive phase and the propagation phase. The different criteria of the initiation and the propagation of these zones have been determined by (Ferdjani and Marigo, 2015) for a crack in a homogeneous material. They can be generalized to the case of an interfacial crack without any difficulty and are presented in the following sections.

3.2.1. Cohesive crack phase $0 < P_0 < \sigma_r$

When $P_0 \ne 0$, a crack must appear in a manner such that the maximal normal stress on the interface remains less than the critical value σ_c. When the load is sufficiently close to 0, the length of the crack is sufficiently small so that u_n is everywhere smaller than the critical value δ_c. Consequently, all the faces of the created crack Γ_d are

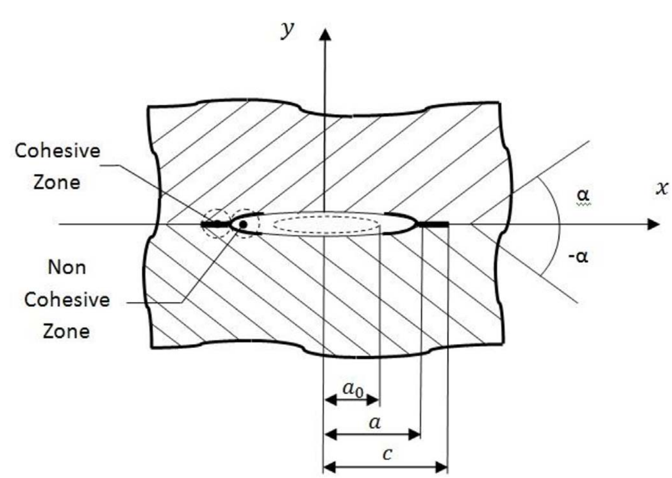

Fig. 2. Geometry of the structure with the initial and created cracks.