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A theory of thermopiezoelectricity with strain gradient and electric field gradient effects

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ABSTRACT

This paper is concerned with a theory of thermopiezoelectricity in which the second gradient of displacement and the second gradient of electric potential are included in the set of independent constitutive variables. First, the basic equations of a linear theory are derived. The field equations for homogeneous and isotropic solids are established and the boundary-initial-value problems are formulated. Then, a uniqueness result for the mixed boundary-initial-value problem and a reciprocity relation are presented. This relation forms the basis of a reciprocal theorem and a new uniqueness result. Finally, the fundamental solutions in the stationary theory and representations of Somigliana type are derived.

1. Introduction

The interaction of electromagnetic fields with elastic solids has been the subject of many investigations (see, e.g., Eringen, 2004 and the literature cited therein). Certain crystals (such as quartz, tourmaline, etc.) when subject to stress, become electrically polarized. This is the simple piezoelectric effect. Conversely, an external electromagnetic field produces deformation in a piezoelectric crystal. We will be concerned with the more general case, in which the electromagnetic field and temperature field are coupled with the deformation field. The theory of thermopiezoelectricity has been studied in various works (see, e.g., Mindlin, 1961; Nowacki, 1986; Eringen and Maugin, 1990).

The second gradient electroelasticity has received in recent years a widespread attention. The origin of the theories of non-simple elastic solids goes back to works of Toupin (1962, 1964), Mindlin (1964), and Mindlin and Eshel (1968). The interest in the gradient theory of elasticity is stimulated by the fact that this theory is adequate to investigate important problems related to size effects and nanotechnology (Askes and Aifantis, 2011). The strain gradient theory and Cosserat theory have been used to study the behaviour of chiral materials (Papanicolopulos, 2011; Ha et al., 2016). The gradient theories of thermomechanics have been studied in various papers (Ahmadi and Firoozbakhsh, 1975; Ieşan and Quintanilla, 1992; Forest et al., 2000, 2002; Ieşan, 2004; Forest and Amestoy, 2008; Forest and Aifantis, 2010; Fernandez-Sare et al., 2010). Forest and Aifantis (2010) have introduced higher order gradients of temperature and concentration to investigate the transport theories. For dielectrics, the polarization gradient theory (Mindlin, 1968) and the electric field gradient theory (Landau and Lifshitz, 1984) are considered as theories for weak nonlocal effects (Maugin, 1979). The electric field gradient theory has been investigated in various papers (Kafadar, 1971; Maugin, 1980; Eringen and Maugin, 1990). Kalpakides and Agiasofitou (2002) have established a theory of electroelasticity including both strain gradient and electric field gradient. Hu and Shen (2009) derived a gradient theory with surface effects for nanosized dielectrics with strain and electric field gradients. A theory of piezoelectricity with rotation gradient effects has been investigated by Wang et al. (2004). Electric field gradient effects have been studied in various papers (see, e.g., Wang et al., 2008; Yang et al., 2006; Yue et al., 2015; Mohammadimehr et al., 2016; Sladek et al., 2017).

In this paper we present a linear theory of thermopiezoelectricity in which the second gradient of displacement and the second gradient of electric potential are included in the set of independent constitutive variables. We have used the equations established by Toupin (1964), Kalpakides and Agiasofitou (2002) and Eringen (2004) to derive the local forms of energy balance and entropy

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production inequality. The restrictions placed on constitutive equations by the second law have been derived following the results presented by Eringen (2004). First, we derive the basic equations of the theory by using the Clausius-Duhem inequality. The field equations for homogeneous and isotropic solids are established and the boundary-initial-value problems are formulated. Then, we establish a uniqueness result for the mixed boundary-initial-value problem. We derive a reciprocity relation which involves two processes at different instants. This relation forms the basis of a reciprocal theorem and a new uniqueness result. The proof of the reciprocal theorem avoids both the use of Laplace transform and the incorporation of the initial conditions in the equations of motion. The new uniqueness result is established without using the definiteness assumptions on the elastic constitutive coefficients. Finally, we establish the fundamental solutions in the stationary theory and derive representations of Somigliana type.

2. Second gradient thermopiezoelectricity

In this section we present the basic equations of a second gradient theory of thermopiezoelectricity. Let us consider a body which at time t_0 occupies the bounded region ∂B of Euclidean three-dimensional space, with Lipschitz boundary ∂B consisting of a finite number of smooth surfaces. Let Γ_p be the intersection of two adjoined smooth surfaces and $C = \cup\Gamma_p$. We assume that B is occupied by a homogeneous solid. Let $\mathbf{u}$ be the displacement vector field. We refer the deformation of the body to a fixed system of rectangular Cartesian axes Ox_k, $(k = 1,2,3)$. We shall employ the usual summation and differentiation conventions: Latin subscripts (unless otherwise specified) are understood to range over the integers $(1,2,3)$ whereas Greek subscripts are confined to the range $(1,2)$, summation over repeated subscripts is implied and subscripts preceded by a comma denote partial differentiation with respect to the corresponding Cartesian coordinate. A superposed dot denotes the material derivative with respect to the time t. We assume that the body is free from initial stresses. The local form of the conservation law of linear momentum may be written in the form

$$t_{ji,j} + \rho f_i = \rho \ddot{u}_i, \tag{1}$$

where t_{ij} is the stress tensor, f_i is the external body force per unit mass, and ρ is the mass density in the reference configuration.

Let g be the density of free charge, $\mathbf{D}$ the electric displacement field, and $\mathbf{E}$ the electric intensity. Then, Maxwell's equations for the quasi-static electric fields can be written as (Eringen and Maugin, 1990)

$$D_{j,j} = g, \quad E_k = -\varphi_{,k}, \tag{2}$$

where φ is the electric potential. The components of surface traction and the classical surface charge density are given respectively by

$$t_i = t_{ji}n_j, \quad G = -D_jn_j. \tag{3}$$

Let ω be an arbitrary material volume in the continuum, bounded by a surface $\partial\omega$ at time t. We suppose that Ω is the corresponding region in the reference configuration B, bounded by a surface $\partial\Omega$. Following Toupin (1964), Kalpakides and Agiasofitou (2002), Wang et al. (2004) and Eringen (2004), we postulate an energy balance in the form

$$\begin{aligned}\int_\Omega \rho(\dot{e} + v_i\dot{v}_i)dv &= \int_\Omega [\rho(f_iv_i + S) + \varphi\dot{g}]dv \\ &+ \int_{\partial\Omega} \left(t_iv_i + m_{ji}v_{i,j} + q + \varphi\dot{G} + E_i\dot{Q}_i\right)da,\end{aligned} \tag{4}$$

for all regions Ω of B and every time, where e is the internal energy per unit mass, v_i are the components of the velocity vector, S is the heat supply per unit mass, q is the heat flux across the surface $\partial\omega$ measured per unit undeformed area, m_{ji} is the hypertraction associated with the surface $\partial\omega$ measured per unit area of $\partial\Omega$, and Q_i is the generalized surface charge density. If we use the relations (1)–(3) and the divergence theorem, then (4) reduces to

$$\begin{aligned}\int_\Omega \rho\dot{e}dv &= \int_\Omega \left(\rho S + t_{ji}v_{i,j} - \varphi_{,i}\dot{D}_i\right)dv \\ &+ \int_{\partial\Omega} \left(m_{ji}v_{i,j} + q - \varphi_{,j}\dot{Q}_j\right)dv,\end{aligned} \tag{5}$$

for all regions Ω and every time. With an argument similar to that used to obtain $(3)_1$, from (5) we get

$$\left(m_{ji} - \mu_{kji}n_k\right)v_{i,j} + q - q_in_i - \left(\dot{Q}_i - \dot{Q}_{ji}n_j\right)\varphi_{,i} = 0, \tag{6}$$

where μ_{kji} is the double stress tensor (Toupin, 1964), q_j is the heat flux vector, and Q_{ji} is the electric quadrupole.

If we use (6) and the divergence theorem, then from (5) we obtain the following local form of energy balance

$$\rho\dot{e} = \tau_{ji}v_{i,j} + \mu_{kji}v_{i,jk} + \rho S + q_{j,j} - \varphi_{,i}\dot{\sigma}_i - \varphi_{,ij}\dot{Q}_{ji}, \tag{7}$$

where

$$\tau_{ji} = t_{ji} + \mu_{kji,k}, \ \ \sigma_i = D_i + Q_{ji,j}. \tag{8}$$

We note that the equation (1) can be written in the form

$$\tau_{ji,j} - \mu_{kji,kj} + \rho f_i = \rho\ddot{u}_i. \tag{9}$$

The equation $(2)_1$ becomes

$$\sigma_{j,j} - Q_{ij,ij} = g. \tag{10}$$

Let us consider a motion of the body which differs from the given motion by a superposed uniform rigid body angular velocity, and assume that $\rho, e, \tau_{ij}, \mu_{kij}, E_i, D_i, Q_{ij}$ and q_i are not affected by such motion (Green and Rivlin, 1964). Then, from (7) we obtain

$$\tau_{ij} = \tau_{ji}. \tag{11}$$

It is known that the skew symmetric part $\mu_{[ij]k}$ makes no contribution to the rate of work over any closed surface in the body. We shall assume that μ_{ijk} is symmetric with respect to i and j. In view of (11) the relation (7) can be written as

$$\rho\dot{e} = \tau_{ij}\dot{e}_{ij} + \mu_{kji}\dot{\kappa}_{kji} + \rho S + q_{j,j} - \varphi_{,i}\dot{\sigma}_i - \varphi_{,ij}\dot{Q}_{ji}, \tag{12}$$

where

$$e_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}), \ \ \kappa_{ijk} = u_{k,ij}. \tag{13}$$

We postulate the entropy production inequality in the form

$$\int_\Omega \rho\dot{\eta}dv - \int_\Omega \frac{1}{T}\rho Sdv - \int_{\partial\Omega} \frac{1}{T}qda \geq 0, \tag{14}$$

for every part Ω of B and every time. Here η is the entropy per unit mass, and T is the absolute temperature which is assumed to be positive. Following Green and Steel (1966), from (14) we obtain