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Analysis of singular stresses at a vertex and along a singular line in three-dimensional bonded joints using a conservative integral

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ABSTRACT

In the present study, singular stress fields in three-dimensional dissimilar material joints are investigated at a vertex and along a free edge (a singular line) of an interface. A conservative integral based on Betti's reciprocal principle is used to calculate the intensities of the singularities in three-dimensional dissimilar material joints. Two geometries in which the angle between the side surface of the vertex equals 60° and 90° and a range of material combinations are used to investigate the characteristics of the stress singularity. The relationships between the angular functions of the stresses at the vertex and along the singular line are studied. The intensities of the singularities at several positions along the singular line of the interface are determined. Then, the relationship between the intensity of the singularity at the vertex and that at a point located on the singular line is investigated. Finally, the intensities of the singularities along the singular line can be described as a function of the distance from the vertex and the singular stress at the vertex.

1. Introduction

Dissimilar material joints or bonded joints have frequently been used in numerous industrial products, especially in electronic components, automobiles, and airplanes. Dissimilar material joints have stress singularities created by mismatches in the material properties across the interface. The stress singularities may lead to fracture and failure in joints. Thus, stress analysis in dissimilar material joints is important for designing joint structures. Generally, it is difficult to accurately calculate stress distributions near the edge of the interface in joints using the conventional finite element method (FEM). Very refined meshes are required to obtain more accurate solutions. Efficient numerical methods are, therefore, required to solve singular stress problems. At the beginning of the development in the numerical analysis, special elements and methods have been developed to solve two-dimensional bonded joint problems, e.g., a hybrid element method (Tong et al., 1973), an enriched FEM (Benzley, 1974; Luangarpa and Koguchi, 2013a,b,c), stress extrapolation (Munz and Yang, 1993), and a conservative integral (Banks-Sills and Sherer, 2002). The above methods confirm that studies on two-dimensional analysis have been conducted. Both the order of the stress singularity and the intensity of the singularity can be determined using several methods, and understanding the stress singularity in two-dimensional joints is clear. However, in engineering applications, a three-dimensional analysis should be considered. From this reason, some studies have been extended to stress analysis in three-dimensional joints.

Singular stress fields in three-dimensional bonded joints are more complicated than those in two-dimensional joints. For three-dimensional cases, the order of the singularity and the intensity of singularity cannot be calculated analytically. In addition, singular stresses are generated not only at the vertices but also along the free edges of the interface. A number of studies have analysed the order of the singularity in three-dimensional bonded joints. For example, Ghahremani (1991) used a numerical variational method, and Pageau and Biggers (1995) used FEM eigenanalysis to calculate the order of the singularity at the vertex. After that, Koguchi (1997) and Koguchi and Muramoto (2000) investigated the order of the singularity both at the vertex and along the singular line. Until now, the order of the singularity at the vertex of the interface in three-dimensional joints was believed to be larger than that in two-dimensional joints.

In addition to the order of the singularity, the intensity of the singularity is also an important factor in studying the characteristics of stress singularity fields. Although there have been some studies

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on the analysis of the intensity of the singularity in three-dimensional dissimilar material joints, e.g., Lee and Im (2003) used the two-state M-integral, Wisessint and Koguchi (2009) developed a three-dimensional enriched FEM, and Koguchi and Da Costa (2010) used a nonlinear least square method, most of these studies analysed only the intensity of the singularity at the vertex.

In the present study, singular stress fields in three-dimensional dissimilar material joints are investigated both at the vertex and along the singular line of the interface. The intensities of the singularities in dissimilar material joints are analysed using the conservative integral based on Betti's reciprocal principle (Stern et al., 1976; Sinclair et al., 1984; Carpenter, 1984). The conservative integral has been extended to analyse two-dimensional dissimilar material joints later (Carpenter and Byers, 1987; Banks-Sills, 1997; Banks-Sills and Sherer, 2002). And then, this method has been applied to predict crack growth near the bimaterial interfaces (Klusak et al., 2012; Kotoul et al., 2012; Profant et al., 2013). This method has proven to be powerful and can accurately calculate the intensity in two-dimensional cases. Over the past ten years, a number of studies have focused on the analysis of the intensity of the singularity in three-dimensional dissimilar material joints. For example, Nomura et al. (2010) analysed the intensity of the singularity at the interfacial corner in an anisotropic bi-material under thermal stress, and Kuo and Hwu (2010) analysed the problem of an anisotropic material interface corner under mechanical loading. However, in most previous studies, the order of the singularity and the angular functions based on two-dimensional analysis were used, which implies that the plane strain condition was assumed in the analysis. Recently, Luangarpa and Koguchi (2014) developed the conservative integral to determine the intensity of the singularity at the vertex of the interface in bi-material joints. After that, this method has been applied to calculate the stress intensity factor along three-dimensional interface cracks (Koguchi et al., 2015). This method uses a finite element approach (FEM eigenanalysis) formulated by Pageau et al. (1996) to calculate the order of the stress singularity and the angular functions of the displacements and stresses. Using three-dimensional FEM eigenanalysis, the orders of the singularity at the vertex and along the singularity line are determined separately. Therefore, the intensity of the singularity can also be calculated separately at the vertex and points along the singular line.

The conservative integral suggested by Luangarpa and Koguchi (2014) is extended to determine the intensity of the singularity along the stress singular line. To our knowledge, no study on the determination of the intensity of the singularity along the stress singular line using the conservative integral has been conducted. In addition, the relationship between the stress singularity at the vertex and along the stress singular line remains unclear. From this reason, the intensities of the singularities at the vertex and at several positions along the singular line of the interface are examined. Material combination of silicon and epoxy, which are usually used in electronic devices, are chosen for this analysis. Two geometries, in which the angle between side surface of the vertex equals 60° and 90°, are investigated. The order of the singularity and angular functions along the singular line of these two models are considered to be the same because the geometries of the side surface are the same. On the other hand, the order of the singularity and the angular functions at the vertex are different depending on the geometry of the vertex. These models are chosen to study an effect of the singularity at the vertex on the singularity along the singularity line. A range of material combinations (five pairs of material combinations) is examined to study the characteristics of the stress singularity in detail. Finally, a relationship between the intensity of the singularity at the vertex and that along the stress singular line is obtained.

2. Analytical formula

In this section, a conservative integral is developed to calculate the intensity of the singularity in the three-dimensional bi-material model, as shown in Fig. 1. This figure indicates that there are two types of stress singularities in the bi-material model:

(1) the stress singularity at the vertex,
(2) the stress singularity at a point on the singular line.

The conservative integral formulation derived by Luangarpa and Koguchi (2014) is used to calculate the intensity of the singularity at the vertex. In addition, the conservative integral formulation for calculating the intensity of the singularity at a point along the singular line is first developed in the present study.

The conservative integral has been developed using Betti's reciprocal principle, as follows:

$$\int_S (T'_i u_i - T_i u'_i) ds = 0 \tag{1}$$

where S is any contour, T_i and T'_i are tractions, and u_i and u'_i are the displacements in the singular and complementary fields, respectively.

Equation (1) is rewritten as an integral with respect to the closed area shown in Fig. 2(a) for the vertex and in Fig. 2(b) for a point on the singular line, as follows:

$$\sum_{j=0}^{n} \int_{S_j} (T'_i u_i - T_i u'_i) ds = 0. \tag{2}$$

The contour, S, is composed of $S = S_0 + S_1 + S_2 + S_3 + S_4 + S_5$ for the vertex in Fig. 2(a) and $S = S_0 + S_1 + S_2 + S_3$ for the point on the singular line in Fig. 2(b), where S_2, S_3, S_4, and S_5 are contours on the free surfaces. Then, the traction is zero on these surfaces. Equation (2) becomes

$$\int_{S_0} (T'_i u_i - T_i u'_i) ds + \int_{S_1} (T'_i u_i - T_i u'_i) ds = 0. \tag{3}$$

The form of the traction is modified to be $T_i = \sigma_{ij}\hat{n}_j$, where $\hat{n}_j$ is the outward unit normal vector to the closed surface, S (see Fig. 3(a)

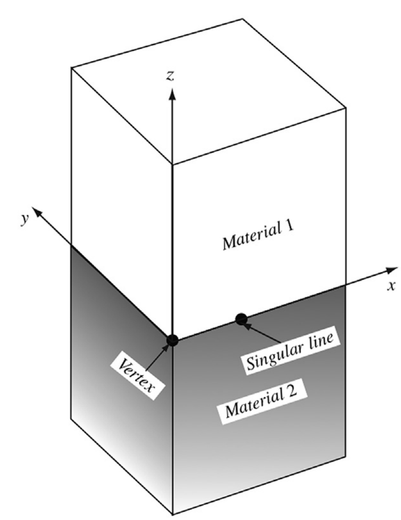

Fig. 1. Three-dimensional dissimilar material joint model.