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Theoretical estimates for flat voids coalescence by internal necking

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ABSTRACT

Coalescence of voids by internal necking is in most cases the last microscopic event related to ductile fracture and corresponds to a localized plastic flow between adjacent voids. Macroscopic load associated to the onset of coalescence is classically estimated based on limit analysis. However, a rigorous upper-bound mathematical expression for the limit-load required for flat voids coalescence that remains finite for penny-shaped voids/cracks is still unavailable. Therefore, based on limit analysis, theoretical upper-bound estimates - both integral expression and closed-form formula - are obtained for the limit-load of cylindrical flat voids in cylindrical unit-cell subjected to boundary conditions allowing the assessment of coalescence, for axisymmetric stress state. These estimates, leading to finite limit-loads for penny-shaped cracks, are shown to be in very good agreement with numerical limit analysis, for both cylindrical and spheroidal voids. Approximate formula is also given for coalescence under combined tension and shear loading. These coalescence criteria can thus be used to predict onset of coalescence of voids by internal necking in ductile fracture modelling.

1. Introduction

Macroscopic ductile fracture of metallic materials has been shown to be related to three different microscopic phenomena, namely void initiation, growth and coalescence (Puttick, 1959). In most cases, debonding or cracking of second-phase particles lead to the creation of voids (Argon et al., 1975; Beremin, 1981; Babout et al., 2004), that grow under mechanical loading (Rice and Tracey, 1969; McClintock, 1968; Marini et al., 1985; Weck and Wilkinson, 2008) until localization appears between adjacent voids leading to void coalescence (Brown and Embury, 1973; Thomason, 1968; Cox and Low, 1974; Weck et al., 2008). Homogeneized macroscopic models of porous materials have been developed since the work of Gurson (Gurson, 1977) based on limit analysis and Rousselier (Rousselier, 1981) based on thermodynamical concepts, and more recently based on non-linear variational principles (Danas and Ponte Castañeda, 2009). Models have been extended to account for void nucleation (Chu and Needleman, 1980), void shape effects (Gologanu et al., 1997; Madou and Leblond, 2012a), plastic anisotropy effects (with or without void effects) (Benzerga and Besson, 2001; Keralavarma and Benzerga, 2010; Monchiet et al., 2008; Morin et al., 2015a) and more recently to single crystals (Han et al., 2013; Paux et al., 2015; Mbiakop et al., 2015). Both microscopic and homogeneized macroscopic models have been assessed through comparisons to computational cell models (see, *e.g.*, (Koplik and Needleman, 1988; Tvergaard, 1990)). Homogeneized models attempt to account for void coalescence, either by considering an empirical acceleration of porosity after a critical value (Tvergaard and Needleman, 1984) or by coupling directly growth model to coalescence model, the latter giving a flow potential after the onset of coalescence (Benzerga et al., 2002). The last method appears to be the most promising as no empirical parameters identification is necessary, but requires accurate void coalescence model which is the main purpose of the present work and will be detailed hereafter. The interested reader is referred to the reviews of Benzerga and Leblond (Benzerga and Leblond, 2010), Besson (Besson, 2010) and Pineau et al. (Pineau et al., 2016) for detailed presentations about ductile fracture.

Experimental observations of void coalescence have distinguished three main mechanisms: *internal necking* where localized plastic flow appears in the intervoid ligament perpendicular to the main loading direction similar to necking observed in tensile test, *void sheeting* involving shear band (Cox and Low, 1974), and *necklace coalescence* where localized plastic flow appears in the intervoid ligament parallel to the main loading direction (Benzerga et al., 2004). In this study, we focus on void coalescence by internal necking, also refers to as coalescence in layers, which is the most common situation. Thomason (Thomason, 1968, 1985a, 1985b,

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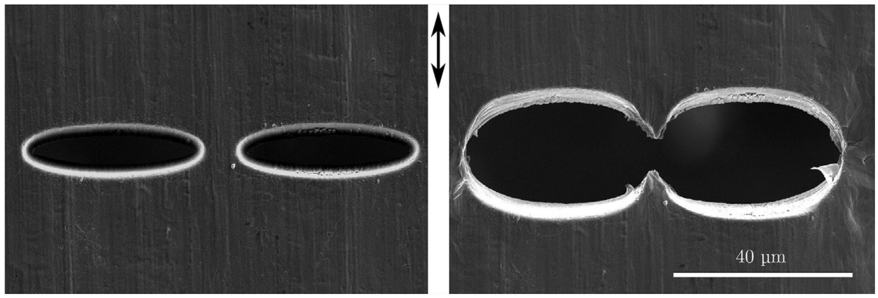

Fig. 1. Experimental observation of flat void coalescence by internal necking. Elliptic cylindrical holes have been drilled through 75μm stainless steel sheet and subjected to uniaxial tension. Black arrow indicates the loading direction.

1990) proposed to assess the *onset of coalescence* through reaching plastic limit-load in the intervoid ligament, considering a perfectly plastic material and making use of both limit analysis and homogeneization technique. The analysis gives the macroscopic stress at which onset of coalescence - which can be seen as a transition from diffuse plastic flow around the void to localized flow in the intervoid ligament with associated elastic unloading in other regions - is expected to occur. Such kinematics have been supported by unit-cell simulations (Koplik and Needleman, 1988). Thomason model have been shown to give quite accurate predictions compared to experimental results (see for example (Weck et al., 2008)), and have been extended to account for strain hardening (Pardoen and Hutchinson, 2000; Scheyvaerts et al., 2011) and secondary voids (Fabrègue and Pardoen, 2008).

The original Thomason model (Thomason, 1990) - and extensions based on it - suffers from two main drawbacks. The first one is that it gives infinite coalescence load in the limit of very flat voids, *i.e.* penny-shaped cracks. Thus, predictions worsen as the void flatness increases. This prevents for example the use of Thomason model in low stress triaxiality conditions, where initially spherical voids tend to form micro-cracks (Tvergaard, 2012). Moreover, although limited experimental results are yet available in literature, evidence of void coalescence of flat voids has been reported for aluminum alloys through Synchrotron Radiation Computed Tomography (SRCT) (Shen et al., 2013). Voids initiate from elongated particles, and remain flat up to coalescence even though plastic deformation occurs because of the small distance between them. Example of flat void coalescence is shown in Fig. 1 on a model experiment. Aluminum alloys are used as structural materials in industrial applications, thus precise estimation of flat void coalescence is needed. The second drawback is that, while the upper-bound theorem of limit analysis was used, Thomason relies at the end on an empirical equation that may not be strictly an upper-bound. Recently, rigorous theoretical upper-bound estimates were obtained by considering cylindrical voids in cylindrical unit-cell (Benzerga and Leblond, 2014; Morin et al., 2015b). To account for finite limit-load of penny-shaped cracks, empirical modification of Thomason model has been proposed in (Benzerga, 2002), while heuristic modifications of rigorous models have been more recently proposed (Torki et al., 2015; Keralavarma and Chockalingam, 2016). However, to the knowledge of the authors, no mathematical expression is available that provides a rigorous upper-bound estimate of the limit-load that remains finite for penny-shaped cracks. Therefore, the aim of this paper is to provide rigorous upper-bound estimates of the limit-load for coalescence of flat voids by internal necking that leads to finite limit-load for penny-shaped cracks, in the case of axisymmetric loading conditions.

In the first part of the paper, cylindrical unit-cell with cylindrical and spheroidal void is described as well as the boundary conditions considered for the study of coalescence. Theoretical results of limit analysis for a von Mises matrix and numerical limit analysis are briefly summarized. In a second part, upper-bound estimates - both integral expression and closed-form formula - for the coalescence limit-load of cylindrical unit-cell containing flat cylindrical void under axisymmetric loading conditions are detailed and compared to supposedly exact numerical results. The estimates are finally compared to numerical results for flat spheroidal voids, and to the predictions of coalescence models with heuristic modifications to account for finite limit-load for penny-shaped cracks (Benzerga, 2002; Torki et al., 2015; Keralavarma and Chockalingam, 2016). In addition, approximate void coalescence criterion for combined tension and shear is proposed based on the coalescence limit-load obtained under axisymmetric loading conditions.

2. Limit analysis of cylindrical unit-cell with voids

2.1. Geometry and boundary conditions

A cylindrical unit-cell Ω of half-height H and radius L containing a coaxial void ω is used in this study (Fig. 2). Two geometries of voids are considered, namely cylindrical (of radius R and half-height h) and spheroidal (of semi-principal length R and h). Two dimensionless ratio will be used in the following:

$$W = \frac{h}{R} \quad \chi = \frac{R}{L} \tag{1}$$

where W is the aspect ratio of the void, and χ the dimensionless length of the inter-void ligament. As only coalescence is studied here, *i.e.* localized plastic flow in the inter-void ligament, the height H is not a parameter of interest (Morin et al., 2015b). The cylindrical unit-cell subjected to the following boundary conditions for the velocity field:

$$\begin{aligned} v_r(L,z) &= D_{11}L \\ v_z(r,\pm H) &= \pm D_{33}H \end{aligned} \tag{2}$$

stands as an approximation of a unit-cell of a periodic array of voids of hexagonal lattice[1] (Fig. 1) under periodic boundary conditions (Koplik and Needleman, 1988) in axisymmetric stress state ($\Sigma_{33} > \Sigma_{11} = \Sigma_{22}$, $\Sigma_{ij} = 0$ for $i \neq j$).

The material is supposed to be rigid-perfectly plastic,[2] obeying von Mises' criterion, and plastic flow is associated by normality. At coalescence, the regions above and below the void unload elastically (Koplik and Needleman, 1988), which thus corresponds in our

[1] The reader is referred to (Kuna and Sun, 1996) for a discussion about the effect of the choice of the unit-cell.

[2] As a classical result of limit analysis that will be used in this study is that elastic strain rates vanish at limit-load.