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On reduced models in nonlinear solid mechanics

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ABSTRACT

The capabilities and limits of current model reduction methods are examined in the case of solid mechanics problems involving significant nonlinearities—such as (visco)plasticity, damage, contact with friction, …—and parameters. Particular emphasis will be put on the PGD method (Proper Generalized Decomposition) and its last developments. These reduced models are the key to the introduction of materials physics in simulation-driven structural design, a domain in which quasi real-time simulations are mandatory.

1. Introduction

1.1. Our main motivation

Our work is part of the “composites” revolution which the aeronautical industry has engaged in, particularly in Europe. The share of composites in civilian planes has grown a lot and vital elements are made of carbon-fiber laminated composites. The current design approach is prohibitively expensive in cost and duration (and, therefore, today inadequate) because it is based almost exclusively on testing. Today, industrializing a new matrix grade requires carrying out the whole series of tests with their numerous stacking sequences all over again, which inhibits innovation.

This is leading the aeronautical industry to a reversal of the situation through “virtual testing”, which consists in replacing, whenever possible, the numerous experimental tests used today by virtual charts (see Fig. 1). This should lead to a significant decrease in the cost and duration of the design and sizing stage. Virtual Charts are particular Reduced Order Models (ROM) associated to goal-oriented quantities. Fig. 2 gives an illustration of a virtual chart for a coupon family of composite plates with a hole, the parameters being the hole diameter as well as the angles defining the fiber directions of the different plies. The virtual chart here gives the maximum of the longitudinal strain in term of the parameters over the parameter set.

1.2. The scope of the paper and the state-of-the-art

This paper deals with the computation offline of a reduced-order model (ROM) in nonlinear solid mechanics. We focus on complex constitutive relations as (visco)plasticity and damage and also unilateral contact with friction and damage. However, we limit ourselves to quasi-static and small displacements conditions. Loading and material parameters could be stochastic and one considers here that they belong to a given set.

We have been working on ROM computation for 30 years with the so-called LATIN-PGD and what we are doing at the present time is the result of many works. PGD means “Proper Generalized Decomposition” and LATIN denotes the computational method which is nonincremental. The LATIN-PGD method was introduced in Ladevèze et al. (Ladevèze, 1985a, 1985b) for viscoplastic materials whose constitutive relations are described using a functional approach. Its extension to modern material descriptions involving internal variables, still for viscoplastic materials, was proposed in Ladevèze et al. (Ladevèze, 1989; Ladevèze, 1991). A number of mathematical properties regarding convergence and error indicators were proved in the book (Ladevèze, 1999). Overview could be found in Ladevèze et al. (Ladevèze, 1999; Chinesta and Ladevèze, 2014; Ladevèze et al., 2009). Originally, PGD was called radial loading approximation, which, to us, meant a “mechanics” approximation in solid mechanics. In 2010, together with F. Chinesta, we changed the name because PGD can be viewed as an extension of the classical Proper Orthogonal Decomposition (POD).

This paper should be seen as a revisit of the LATIN-PGD leading to a new, general and robust PGD computation technique. This is

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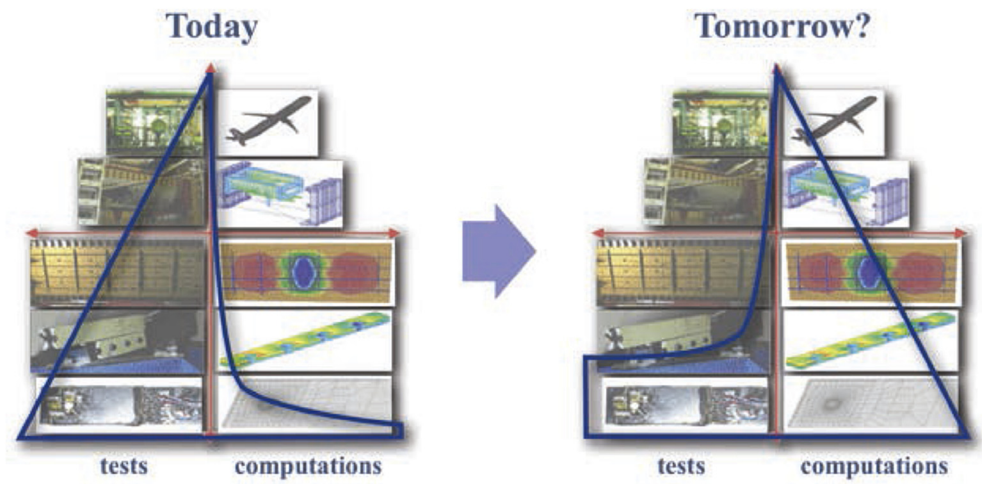

Fig. 1. Future of the test/computation pyramid.

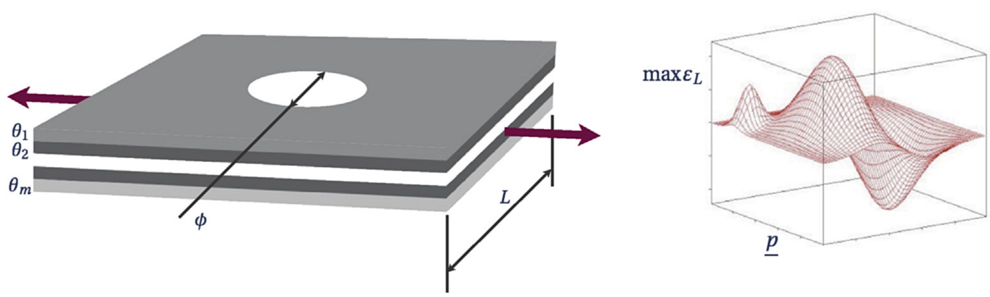

Fig. 2. Virtual chart for a composite coupon family.

based on an "abstract" reformulation of parametric nonlinear solid mechanics problems defined over a time-space domain. This work is the follow-up to (Ladevèze et al., 2010; Relun et al., 2013; Allier et al., 2015; Jessus et al., 2016). Today, there are few other works except the works of Ryckelynck and his group done with POD (Ryckelynck, 2005, 2009; Ryckelynck et al., 2012). However, things are changing and today there are more and more POD-approaches developed in relation with the homogenization technique FE^2 (Lamari et al., 2010; Ammar et al., 2007; Hernández et al., 2014; Radermacher and Reese, 2015; Yvonnet et al., 2013). Another attempt has been done recently with the Asymptotic Numerical Method (ANM) but this approach seems to be quite limited. PGD, like any ROM technique for nonlinear problems, can lead to the computation of numerous nonlinear integrals, which can become very expensive. Additional reduction or interpolation are then introduced to reduce their computation cost. This is done offline with the PGD computation. Our answer is the so-called Reference Point Method (RPM), which is recalled here; furthermore, a performance analysis is given in term of the RPM-parameters.

1.3. Basic features of the new generation of ROM computational methods

By new generation of ROM computational methods, we intend Reduced Based method (RB) (Patera and Rozza, 2006; Rozza and Veroy, 2007; Maday and Ronquist, 2004; Barrault et al., 2004), POD (Kunisch and Xie, 2005; Lieu et al., 2006; Gunzburger et al., 2007) and PGD methods. They are based on the same ideas. The first and main idea is that the shape functions are not as usual a priori given. They are computed simultaneously with the solution itself thanks to an iterative procedure. For a problem defined over the time-space domain, the solution is then written as:

$$\mathbf{s}(t,\boldsymbol{x}) \approx \mathbf{s}^m(t,\boldsymbol{x}) = \sum_{i=1}^{m} a_i \boldsymbol{\Psi}_i(t,\boldsymbol{x}) \tag{1}$$

The second idea is to introduce a variable separation hypothesis or something equivalent:

$$\boldsymbol{\Psi}_i(t,\boldsymbol{x}) = \lambda_i(t)\boldsymbol{\Gamma}_i(\boldsymbol{x}) \tag{2}$$

where the time functions λ_i and the space functions $\boldsymbol{\Gamma}_i$ are arbitrary. That is a departure from other approximation methods for which shape functions are a priori given or partially given. However, ROM computational techniques are quite different (Chinesta and Ladevèze, 2014).

The PGD is characterized by two ingredients: the variable separation hypothesis and a residual which should be minimized or something equivalent. Let R be the residual which satisfies:

$$\forall \mathbf{s} \in \mathbf{S}^{[0,T]},\ R(\mathbf{s}) \geqslant 0 \quad \text{and} \quad R(\mathbf{s}) = 0 \Rightarrow \mathbf{s} = \mathbf{s}_{ex} \tag{3}$$

the problem to solve is then:

$$\min_{\mathbf{s}^m \in \mathbf{S}^{[0,T]}} R(\mathbf{s}^m) \tag{4}$$

which is twice nonlinear. Mechanically, it is nonlinear. Furthermore, the computation of a PGD approximation is always a nonlinear problem.