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Three-dimensional stress analysis of orthotropic curved tubes-part 1: Single-layer solution

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ABSTRACT

In this paper (Part 1), a new displacement-based method is proposed to investigate orthotropic curved tubes subjected to pure bending moment. A displacement approach of Toroidal Elasticity is chosen to analyze orthotropic curved tubes with a single layer. The governing equations are developed in three toroidal coordinates system. The method of successive approximation is used to find the general solution. Then, the governing equations are decomposed into different orders, based on a small parameter. The formulas for calculating different order displacement components are derived. The accuracy of the proposed method is subsequently verified by comparing the numerical results obtained using the proposed method with finite element method (FEM), stress-based Toroidal Elasticity and Lekhnitskii solution. The results show good correspondence. The proposed method provides advantages in terms of computational time compared to FEM.

1. Introduction

Composite tubes are structures that are frequently used in the aerospace, offshore and infrastructure industries. Prediction of the state of stress and strain in different layers of composite tubes is of theoretical interest and practical importance. In all applications, accurate design and inclusive analysis are important to guarantee safety. It should be noted that a stress analysis of composite cylindrical structures is often a complex task. A few reasons are responsible for such a complexity such as: governing equations of composite tubes. In addition, the curved tube geometry is a lot more complicated than a flat geometry. Composite shells and tubes have been investigated by many researchers. The following two sections are a literature review of composite shell and tube analyses, and are based on whether structures have free-edges (*i.e.,* shells) or closed cross sections (*i.e.,* tubes).

1.1. Shell analysis

To obtain a prediction of structural response, a third-order shell theory was proposed by Huang (1994) based on Reddy's parabolic shear strain distribution. The theoretical modeling of laminated composite shells with arbitrary shapes was developed to estimate shear stresses and avoid shear correction factors (Ossadzow et al., 1995). In a similar way, Di and Rothert (1995) obtained stress fields in orthotropic cylindrical shells. Elasticity solutions for other types of loading of homogeneous composite shells were summarized in Kardomateas (1996). Zhang et al. (1997) presented an analytical method to obtain interlaminar stresses at curved boundaries of symmetric composite shells under in-plane loading on the basis of a zero-order approximation of the boundary-layer theory. Khare et al. (2003) analyzed thermo-mechanical behavior of simply supported cross-ply composite and sandwich laminated cylindrical and spherical doubly curved shells. A finite element model was used by Hossain (2004) to study stresses of thick composite doubly curved shells. Kress et al. (2005) proposed a finite element model, which diminished the number of free parameters for each layer, to determine interlaminar stress distributions in laminated singly curved structures. Oktem and Chaudhuri (2007) used a higher order shear deformation theory to obtain an analytical solution for the deformation of a finite-dimensional cross-ply doubly-curved panel. Interlaminar normal stress distributions in moderately thick singly curved laminates were obtained to predict the critical delamination loads observed in experiments (Roos et al., 2007a, 2007b).

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1.2. Tube analysis

Lekhnitskii (1981) developed elasticity solutions for monolithic homogeneous orthotropic cylindrical tubes. The case of uniform external pressure was developed by Kardomateas (1993). Herakovich (1998) provided extensive coverage of laminated composite tubes for axial, torsional, pressure and thermal loading with aspect ratio studies. To find out the energy absorption characteristics of glass-fiber circular tubes, a study was performed by Pickett and Dayal (2012). A method was developed to analyze the pure bending of arbitrary laminated composite tubes (Zhang et al., 2014). They verified formulations with FEM results obtained using ABAQUS. Menshykova and Guz (2014) performed a stress analysis on thick laminated composite tubes subjected to bending load. They found stresses as a function of the material properties, thickness, lay-up and the magnitude of loads. Capela et al. (2015) investigated the fatigue behavior of composite tubes under bending/torsion dynamic loadings. Recently, Nowak and Schmidt (2015) compared some methods to study fiber metal laminated cylinders under an axisymmetric load. A developed theoretical model was validated by FEM analysis. Jonnalagadda et al. (2015) presented an analytical model for a special design of thin composite tubes subjected to combined bending and torsion. They verified the theoretical results with FEM analysis.

The above review shows that there has been no work done to predict stress distributions in composite curved tubes subjected to mechanical loadings. Although finite element methods can be used for analyzing such structures, it is necessary to do the meshing for each structure every time some dimensions or lay-up sequences are changed. Therefore, it is desired to have a method where inputs to obtain solutions are simple; i.e. one only needs to enter in the actual dimensions or lay-up sequences without re-meshing work. The present work is devoted to develop a method that can provide stresses, strains and deformations for composite curved tubes subjected to pure bending moment based on simple inputs. Displacement approach of Toroidal Elasticity (DBTE) and successive approximation method are used. Comparison is made between results obtained for the proposed method with finite element method (FEM), stress-based Toroidal Elasticity (SBTE) and Lekhnitskii solution.

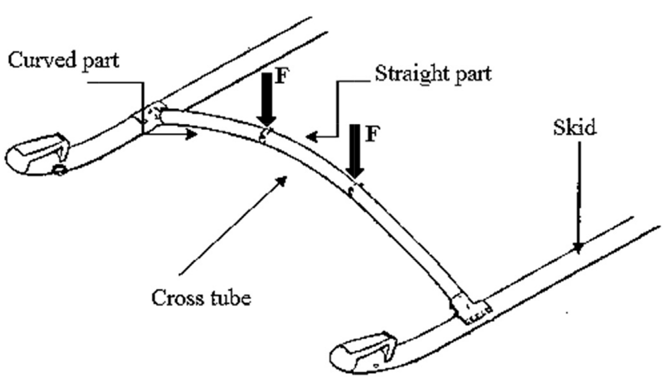

Fig. 1. Helicopter landing gear.

2. Motivation

The conventional landing gears for helicopters consist of two parallel curved cross tubes, which are connected by two longitudinal skid tubes as seen in Fig. 1. The cross tube of the helicopter landing gear consists of straight and curved tubes. Derisi (2008) designed and manufactured composite straight tubes and performed four-point bending tests to determine the strains to failure of different laminates. Recently, a method for the stress analysis of thick composite straight tubes subjected to cantilever loading was developed (Yazdani Sarvestani et al., 2016a, 2016b). Now, in order to provide some insight into the mechanical behavior of the curved part of the helicopter landing gear, a simple-input displacement-based method is developed to examine stresses in a composite curved tube.

3. Displacement field of a single-layer composite curved tube

Toroidal Elasticity (TE) is a three-dimensional theory used for the elastostatic analysis of thick curved tubes. Although earlier studies on TE were performed by a few researchers, Lang (1984) made a major improvement on the development of TE. This researcher employed specifically a stress approach, obtaining solutions and satisfying the equilibrium equations. Zhu and Redekop (1994) developed a displacement approach of TE for isotropic materials. Solutions satisfied Navier equations. Such an approach has the advantage of yielding the displacements, as well as stresses. In this study, the displacement field of a single-layer composite curved tube is derived using a displacement approach of Toroidal Elasticity (DBTE) and the method of successive approximation.

3.1. Governing equations in Toroidal coordinates

A single-layer orthotropic curved tube with a bend radius R is subjected to a pure bending moment, M, acted in the plane of $\phi = 90^\circ$ as shown in Fig. 2. Annular cross section is bounded by radii a and b. Toroidal coordinate system (r, ϕ, θ) is placed at the mid-span of the composite curved tube where r and ϕ are polar

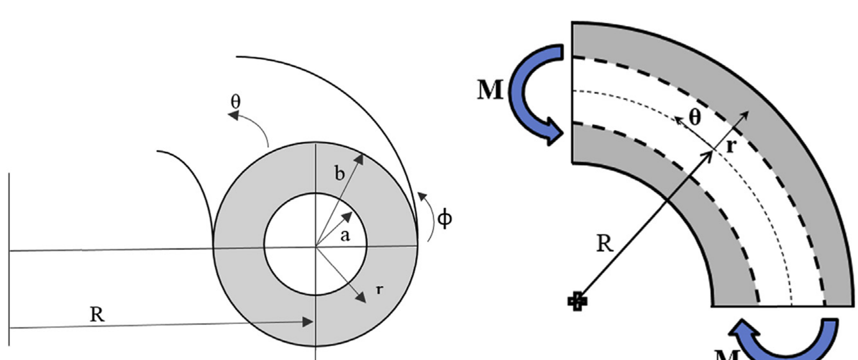

Fig. 2. Geometry and coordinate system of the composite curved tube.

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