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Three-dimensional stress analysis of orthotropic curved tubes-part 2: Laminate solution

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ABSTRACT

The study described in this paper (Part 2) presents a new simple-input method to study thick laminated composite curved tubes subjected to mechanical loadings. First, a displacement approach of Toroidal Elasticity was chosen to obtain the displacement field of single-layer composite curved tubes (Part 1). Then, a layerwise method is employed to develop the most general displacement field of elasticity for arbitrary laminated composite curved tubes. The principle of minimum total potential energy is applied to calculate stresses in thick composite curved tubes under pure bending moment. The accuracy of the proposed method is subsequently verified by comparing the numerical results obtained using the proposed method with finite element method (FEM), experimental data and a solution available in the literature. The results show good correspondence. Also, the proposed method provides advantages in terms of computational time compared to FEM.

1. Introduction

Composite tubes are structures that are frequently used in aerospace, offshore and infrastructure industries. These structures usually have thin or moderately thick walls and are subjected to certain loads such as tension, torsion, shear and bending. Prediction of the state of stress and strain in different layers of composite tubes is of theoretical interest and practical importance. In all applications, accurate design and inclusive analysis are important to guarantee safety. It should be noted that a stress analysis of cylindrical composite structures is often a complex task. A few reasons are responsible for such a complexity such as: governing equations of composite tubes and the layerwise failure of composite materials. In addition, the curved tube geometry is a lot more complicated than flat geometries. Composite straight and curved tubes have been investigated by many researchers.

1.1. Straight beams and tubes

Lekhnitskii (1981) developed elasticity solutions for monolithic homogeneous orthotropic cylindrical shells. A finite difference solution was presented for elastic wave propagation in laminated composite hollow tubes under axial plane strain (Larom et al., 1991). The effects of lay-up sequence, ply angle and stiffnesses on displacement and stress distributions were investigated. Kollár and Springer (1992) studied the stress analysis of composite cylinders and cylindrical segments subjected to hygrothermal and mechanical loads. The case of uniform external pressure and orthotropic homogeneous material was developed by Kardomateas (1993). Three-dimensional stress and displacement analyses of transversely loaded for laminated hollow cylinders with cross-ply lay-up were investigated (Ye and Soldatos, 1994). To find out the energy absorption characteristics of glass-fiber circular tubes, a study was performed by Pickett and Dayal (2012). Sun et al. (2014) performed a general stress analysis for anisotropic composite hollow cylindrical structures subjected to different loads. A method was developed to analyze the pure bending of arbitrary laminated composite tubes (Zhang et al., 2014). They verified formulations with FEM results obtained using ABAQUS. Menshykova and Guz (2014) performed a stress analysis on thick laminated composite pipes subjected to bending loads. They found stresses as a function of the material properties, thickness, lay-up and the magnitude of load. Recently, static analysis of carbon nanotube-reinforced composite cylinder under thermo-mechanical was studied using Mori-Tanaka theory (Arani et al., 2015). Nowak and Schmidt (2015) compared some methods to study fiber metal laminate cylinders

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under an axisymmetric load. A developed theoretical model was validated by FEM analysis. Jonnalagadda et al. (2015) presented an analytical model for a special design of thin composites tubes subjected to combined bending and torsion. They verified the theoretical results with FEM analysis.

1.2. Curved beams and tubes

Qatu (1993) analyzed thin and moderately thick laminated composite curved beams to find natural frequencies. Shear and radial stresses in curved beams were derived based on satisfying both equilibrium equations and static boundary conditions on the surfaces of beams (Yu and Nie, 2005). Dryden (2007) obtained stress distributions across a functionally graded circular beam subjected to pure bending by using stress functions. The free vibration analysis was performed on functionally graded beams with curved axis by using the finite element method to discretize the motion equations (Piovan et al., 2012). A first order shear deformation theory was used to study static and free vibration behavior of generally laminated curved beams (Hajianmaleki and Qatu, 2012). Wang and Liu (2013) presented elasticity solutions for curved beams with orthotropic functionally graded layers subjected to a uniform load on the outer surface by means of Airy stress function method. A mathematical model was developed to analyze behavior of laminated glass curved beams (Aşık et al., 2014).

The above review shows that there is a need of developing a higher-order simple-input method to analyze thick laminated composite curved tubes subjected to different mechanical loading conditions. Although finite element methods can be used for analyzing such structures, it is necessary to do the meshing for each structure every time some dimensions or lay-up sequences are changed. Therefore, it is desired to have a method where inputs to obtain solutions are simple; i.e. one only needs to enter in the actual dimensions or lay-up sequence without re-meshing work. The present work is devoted to develop a method that can predict stresses, strains and deformations in thick composite curved tubes subjected to pure bending moment with simple inputs. Displacement approach of Toroidal Elasticity (TE) and a layerwise method are used. Comparison is made between results obtained for the proposed procedure with experimental data, FEM and a solution available in the literature.

2. Formulation

The displacement field of single-layer composite curved tubes was derived using Toroidal Elasticity and the method of successive approximation (Yazdani Sarvestani et al., 2016). Now, by developing the displacement field of thick laminated composite curved tubes based on the displacement field of single-layer composite curved tubes using a layerwise method, a new displacement-based method is proposed to analyze stresses in composite curved tubes subjected to pure bending moment.

2.1. Displacement field of laminated composite curved tubes

A thick laminated orthotropic curved tube with a bend radius R, mean radius R_1 and thickness h is subjected to pure bending moment, M, as shown in Fig. 1a. Annular cross section is bounded by radii a and b. Toroidal coordinate system (r, ϕ, θ) is placed at the mid-span of the composite curved tube where r and ϕ are polar coordinates in the plane of the curved tube cross section and θ defines the position of the tube cross section.

Based on the developed displacement components in (Yazdani Sarvestani et al., 2016), the general form displacement field of single-layer composite curved tubes of the kth plane and up to the nth order is presented as (the detailed derivation can be found in (Yazdani Sarvestani et al., 2016)):

$$
\begin{aligned}
U^{(k)}(r,\phi,\theta) &= \sum_{i=0}^{n} \varepsilon^i B_i^{(k)}(r)\cos(i\phi)\cos(\theta)\\
V^{(k)}(r,\phi,\theta) &= \sum_{i=0}^{n} \varepsilon^i A_i^{(k)}(r)\sin(i\phi)\cos(\theta)\\
W^{(k)}(r,\phi,\theta) &= \sum_{i=0}^{n} \varepsilon^i C_i^{(k)}(r)\cos((i-1)\phi)\sin(\theta)
\end{aligned}
\tag{1}
$$

where

$$
\begin{aligned}
B_i^{(k)}(r) &= \Big(a_i^{(k)} B_{1,m_i^{(k)}} r^{m_i^{(k)}} + b_i^{(k)} B_{1,m_i'^{(k)}} r^{m_i'^{(k)}} + c_i^{(k)} B_{1,-m_i^{(k)}} r^{-m_i^{(k)}}\\
&\quad + d_i^{(k)} B_{1,-m_i'^{(k)}} r^{-m_i'^{(k)}}\Big)\\
A_i^{(k)}(r) &= -\Big(a_i^{(k)} A_{2,m_i^{(k)}} r^{m_i^{(k)}} + b_i^{(k)} A_{2,m_i'^{(k)}} r^{m_i'^{(k)}} + c_i^{(k)} A_{2,-m_i^{(k)}} r^{-m_i^{(k)}}\\
&\quad + d_i^{(k)} A_{2,-m_i'^{(k)}} r^{-m_i'^{(k)}}\Big)\\
C_i^{(k)}(r) &= \Big(e_i^{(k)} r^{\overline{m}_i^{(k)}} + f_i^{(k)} r^{-\overline{m}_i^{(k)}}\Big)
\end{aligned}
\tag{2a}
$$

$$
\begin{aligned}
\overline{m}_n^{(k)} &= \pm i\left(\frac{\overline{C}_{66}^{(k)}}{\overline{C}_{55}^{(k)}}\right)^{\frac{1}{2}}\\
B_{1,m_i^{(k)}} &= -\left(\overline{C}_{22}^{(k)} + \frac{1}{2}\overline{C}_{44}^{(k)}\right) + \frac{i}{(i+1)} m_i^{(k)}\left(\frac{1}{2}\overline{C}_{44}^{(k)} + \overline{C}_{12}^{(k)}\right)\\
A_{2,m_i^{(k)}} &= -\left(\overline{C}_{22}^{(k)} + \frac{1}{2}\overline{C}_{44}^{(k)}\right) - \frac{i}{(i+1)} m_i^{(k)}\left(\frac{1}{2}\overline{C}_{44}^{(k)} + \overline{C}_{12}^{(k)}\right)
\end{aligned}
\tag{2b}
$$

and $m_i^{(k)}, -m_i^{(k)}, m_i'^{(k)}$ and $-m_i'^{(k)}$ are the 4 roots of the following equation:

$$
\begin{aligned}
&\left(\frac{1}{2}\overline{C}_{44}^{(k)}\overline{C}_{11}^{(k)}\right) m_i^{(k)4} + \Big(-i^2\overline{C}_{11}^{(k)}\overline{C}_{22}^{(k)} - \frac{1}{2}\overline{C}_{11}^{(k)}\overline{C}_{44}^{(k)} - \frac{1}{2}\overline{C}_{44}^{(k)}\overline{C}_{22}^{(k)}\\
&\quad + i^2\overline{C}_{12}^{(k)}\overline{C}_{44}^{(k)} + i^2\overline{C}_{12}^{(k)2}\Big) m_i^{(k)2} + \frac{i^4 - 2i^2 + 1}{2}\overline{C}_{22}^{(k)}\overline{C}_{44}^{(k)} = 0
\end{aligned}
\tag{2c}
$$

with i being the order number from 0 to the nth order (1, 2, 3, …). $\overline{C}_{ij}^{(k)}$ represent the off-axis stiffnesses. Also, n and k present the order number (i.e., $n = 0, 1, 2, 3, 4, \ldots$) and layer number (i.e., $k = 1, 2, \ldots, N$), respectively.

2.2. Layerwise theory (LWT)

The equivalent single-layer theories are not able to precisely find stresses and strains in laminated composites. But, LWT allows each layer of the laminate to act like a real three-dimensional layer while being able to present good results for the local quantities. In LWT, the displacement components of a generic point in the laminate are assumed as (Yazdani Sarvestani and Yazdani Sarvestani, 2011; 2012):

$$
\begin{aligned}
U(z,\phi,\theta) &= u_k(\phi,\theta)\Omega_k(z)\\
V(z,\phi,\theta) &= v_k(\phi,\theta)\Omega_k(z)\\
W(z,\phi,\theta) &= w_k(\phi,\theta)\Omega_k(z) \quad (k = 1,2,\ldots,N+1)
\end{aligned}
\tag{3}
$$

with k, here and in what follows, being a dummy index implying summation of terms from $k = 1$ to $k = N+1$. Note that z is the local direction starting from the mid-thickness of the curved tube cross section (see Fig. 1b). The variable N corresponds to the total number