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Small-scale indentation of a hemispherical inhomogeneity in an elastic half-space

Ivan I. Argatov, Federico J. Sabina*

Instituto de Investigaciones en Matemáticas Aplicadas y en Sistemas, Universidad Nacional Autónoma de México, Apartado Postal 20-726, Delegación Alvaro Obregon, 01000, México, D. F., Mexico

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ABSTRACT

The axisymmetric problem of small-scale frictionless indentation of an elastic hemispherical inhomogeneity embedded at the free surface of a semi-infinite elastic matrix is considered. It is assumed that the radius of contact area is relatively small compared with the radius of the inhomogeneity. The first-order asymptotic model for the incremental indentation stiffness is presented in terms of the coefficient of local compliance, which is evaluated based on the analytical solution for the surface Green's function. The influence of both Poisson's ratios on the corresponding indentation scaling factor, which reflects the effect of localized inhomogeneity, is studied in detail.

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1. Introduction

Indentation technique (Hardy et al., 1971; Follansbee and Sinclair, 1984; Hill et al., 1989) has been proved very useful in testing mechanical properties of small material samples and, in particular, thin films (Antunes et al., 2007; Hemmouche et al., 2013). By modeling the sample configuration as an elastic layer bonded to a rigid base or to an elastic substrate, one can study the corresponding thickness effect (Hayes et al., 1972; Argatov et al., 2013) or the substrate effect (Yu et al., 1990; Gao et al., 1992; Chen and Vlassak, 2001; Perriot and Barthel, 2004; Argatov and Sabina, 2014).

In recent years, the AFM indentation tests have been applied for characterizing composite materials (Gregory and Spearing, 2005; Constantinides et al., 2006), for which new identification methods should be developed in order to take into account the effect of material inhomogeneity (Kabele et al., 2008).

In particular, an important sample geometry is represented by a hemispherical inhomogeneity embedded at the free surface of an elastic half-space made of another material (Fig. 1a). Axially-symmetric finite-element solutions of the indentation problems for an elastic hemispherical inhomogeneity were obtained by Batog et al. (2008) and Kabele et al. (2008) under simplifying assumptions that the values of Poisson's ratio is assumed to be the same in the hemispherical inhomogeneity and the semi-infinite matrix.

In the present paper, we develop the first-order asymptotic model for the incremental indentation stiffness, which in the axisymmetric case can be written as follows (Argatov, 2010; Argatov and Sabina, 2014):

$$\frac{dP}{dw} \simeq \frac{2aE_{\rm eff}}{1-\varepsilon\frac{2a_0}{\pi}}. \qquad (1)$$

Here, P is the contact force, w is the indenter displacement, $\varepsilon = a/l$ is a small parameter, a is the radius of contact area, l is the radius of inhomogeneity, $E_{\rm eff}$ is the effective elastic modulus defined through the formula

$$\frac{1}{E_{\rm eff}} = \frac{1-\overline{\nu}}{2\overline{G}} + \frac{1-\nu_0^2}{E_0}. \qquad (2)$$

Recall that the effective modulus $E_{\rm eff}$ is used to account for the effect of elastic deformation of the indenter, whose Young's modulus and Poisson's ratio are denoted by E_0 and ν_0, respectively. Formula (1) contains the so-called (Argatov, 2002) coefficient of local compliance a_0, which bears information about the inhomogeneity geometry, the interface conditions between the inhomogeneity and matrix, and depends on Poisson's ratios $\overline{\nu}$ and ν as well as on the inhomogeneity-matrix shear moduli ratio $\Gamma = \overline{G}/G$.

* Corresponding author.
E-mail addresses: ivan.argatov@gmail.com (I.I. Argatov), fjs@mym.iimas.unam.mx (F.J. Sabina).

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2. The hemispherical inhomogeneity subjected to a concentrated force

In this section, we briefly outline the analytical solution obtained by Tsuchida et al. (1990) for the three-dimensional mixed boundary value problem of a hemispherical inhomogeneity, ω, embedded at the free surface of an elastic half-space, $\mathbb{R}^3_+ = \{\mathbf{x} = (x_1, x_2, x_3) : x_3 > 0\}$, and subjected to a concentrated force, P. By the way, we correct misprints and shortcomings in the mentioned study.

We assume that the inhomogeneity ω is perfectly bonded to the semi-infinite medium, and the continuity and equilibrium conditions along the interface $\gamma = \partial\omega\cap\mathbb{R}^3_+$ are formulated as follows:

$$\overline{\mathbf{u}}(\mathbf{x}) = \mathbf{u}(\mathbf{x}), \quad \overline{\boldsymbol{\sigma}}^{(n)}(\mathbf{x}) = \boldsymbol{\sigma}^{(n)}(\mathbf{x}), \quad \mathbf{x}\in\gamma. \tag{3}$$

Here, $\overline{\mathbf{u}}$ and $\overline{\boldsymbol{\sigma}}^{(n)}$, $\mathbf{u}$ and $\boldsymbol{\sigma}^{(n)}$ are the displacement and stress vectors in the inhomogeneity ω and in the matrix $\mathbb{R}^3_+\backslash\overline{\omega}$, respectively.

Let us denote the systems of cylindrical and spherical coordinates by (r,θ,z) and (R,θ,ϕ) respectively (see Fig. 1). Then, in the spherical coordinates, the boundary conditions (3) can be rewritten as

$$(\overline{u}_R)_{R=l} = (u_R)_{R=l}, \quad (\overline{u}_\phi)_{R=l} = (u_\phi)_{R=l}, \tag{4}$$

$$(\overline{\sigma}_R)_{R=l} = (\sigma_R)_{R=l}, \quad (\overline{\tau}_{R\phi})_{R=l} = (\tau_{R\phi})_{R=l}. \tag{5}$$

Note that in the axisymmetric case, $\overline{u}_\theta = u_\theta = 0$ and $\overline{\tau}_{R\theta} = \tau_{R\theta} = 0$.

$$\begin{aligned}
\sigma_R &= \frac{\partial^2\Phi_0}{\partial R^2} + \mu R\frac{\partial^2\Phi_3}{\partial R^2} - 2(1-\nu)\mu\frac{\partial\Phi_3}{\partial R} - 2\nu\frac{(1-\mu^2)}{R}\frac{\partial\Phi_3}{\partial\mu},\\
\sigma_\theta &= \frac{1}{R}\frac{\partial\Phi_0}{\partial R} - \frac{\mu}{R^2}\frac{\partial\Phi_0}{\partial\mu} + (1-2\nu)\mu\frac{\partial\Phi_3}{\partial R} - \Big[2\nu + (1-2\nu)\mu^2\Big]\frac{1}{R}\frac{\partial\Phi_3}{\partial\mu},\\
\sigma_\phi &= \frac{1-\mu^2}{R^2}\frac{\partial^2\Phi_0}{\partial\mu^2} - \frac{\mu}{R^2}\frac{\partial\Phi_0}{\partial\mu} + \frac{1}{R}\frac{\partial\Phi_0}{\partial R} + \frac{\mu(1-\mu^2)}{R}\frac{\partial^2\Phi_3}{\partial\mu^2} + (1-2\nu)\mu\frac{\partial\Phi_3}{\partial R} - \Big[1 + (1-2\nu)\Big(1-\mu^2\Big)\Big]\frac{1}{R}\frac{\partial\Phi_3}{\partial\mu},\\
\tau_{R\phi} &= \sin\phi\left(\frac{1}{R^2}\frac{\partial\Phi_0}{\partial\mu} - \frac{1}{R}\frac{\partial^2\Phi_0}{\partial R\partial\mu} + (1-2\nu)\frac{\partial\Phi_3}{\partial R} - \mu\frac{\partial^2\Phi_3}{\partial R\partial\mu} + 2(1-\nu)\frac{\mu}{R}\frac{\partial\Phi_3}{\partial\mu}\right),\\
\tau_{\phi\theta} &= \tau_{R\theta} = 0.
\end{aligned} \tag{10}$$

2.1. Green's function and asymptotic coefficients of local compliance

At the coordinate center, O, the vector-function $\overline{\mathbf{u}}$ should satisfy the following asymptotic condition:

$$\overline{\mathbf{u}}(\mathbf{x}) = P\overline{\mathbf{T}}(\mathbf{x}) + O(1), \quad \mathbf{x}\to O. \tag{6}$$

Here, $\overline{\mathbf{T}}$ is the solution of the Boussinesq problem for an elastic half-space comprised of the same material properties as the inhomogeneity.

In the case of a unit force P, the solution to the elastic problem (3), (6) represents Green's vector-function with a pole at the point O. The structure of the next terms in the asymptotic expansion (6) was studied in detail by Argatov (2002). It was shown that in asymptotic analysis of frictionless contact problems, the following asymptotic expansion plays an important role:

$$\frac{2\pi\overline{G}}{(1-\overline{\nu})P}\overline{u}_3(x_1,x_2,0) = \frac{1}{\sqrt{x_1^2+x_2^2}} - \frac{1}{l}\sum_{n=0}^{\infty}\frac{a_n}{l^{2n}}\Big(x_1^2+x_2^2\Big)^n. \tag{7}$$

Here, $\overline{G}$ and $\overline{\nu}$ are the shear modulus and Poisson's ratio of the inhomogeneity, l is the radius of the inhomogeneity. The asymptotic constants a_0,a_1,... are called the coefficients of local compliance (Argatov, 2002). We emphasize that the local-compliance coefficients are dimensionless and depend on the inhomogeneity-matrix shear moduli ratio

$$\Gamma = \frac{\overline{G}}{G} \tag{8}$$

as well as on Poisson's ratios $\overline{\nu}$ and ν, where G and ν are the shear modulus and Poisson's ratio of the semi-infinite matrix.

2.2. Boussinesq potentials

In the case of axisymmetry about the x_3-axis, the general solution to the elasticity equations in the matrix is given by the Boussinesq potentials

$$\begin{aligned}
2Gu_R &= \frac{\partial\Phi_0}{\partial R} + \mu\left(R\frac{\partial\Phi_3}{\partial R} - (3-4\nu)\Phi_3\right), \quad u_\theta = 0,\\
2Gu_\phi &= -\frac{\sin\phi}{R}\frac{\partial\Phi_0}{\partial\mu} + \sin\phi\left(-\mu\frac{\partial\Phi_3}{\partial\mu} + (3-4\nu)\Phi_3\right),
\end{aligned} \tag{9}$$

where Φ_0 and Φ_3 are harmonic functions, i. e., $\nabla^2\Phi_0 = \nabla^2\Phi_3 = 0$,

$$\nabla^2 = \frac{1}{R^2}\frac{\partial}{\partial R}\left(R^2\frac{\partial}{\partial R}\right) + \frac{1}{R^2}\frac{\partial}{\partial\mu}\left(\left(1-\mu^2\right)\frac{\partial}{\partial\mu}\right), \quad \mu = \cos\phi.$$

The corresponding stress field is given by[1]

The solution $\overline{\mathbf{T}}$ of the Boussinesq problem is expressed in terms of the Boussinesq potentials as

$$\overline{\Phi}_0^0 = -\frac{p_0 l^2}{2}(1-2\overline{\nu})\ln(R+x_3), \quad \overline{\Phi}_3^0 = -\frac{p_0 l^2}{2}\frac{1}{R}, \tag{11}$$

where p_0 is the equivalent pressure given by

$$p_0 = \frac{P}{\pi l^2}, \tag{12}$$

and the quantities denoted by a bar refer to the inhomogeneity.

[1] Formulas (10) coincide with the corresponding relations used by Tsuchida et al. (1990) except for the formula for σ_ϕ.

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