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An analytical study on the buckling and free vibration of rectangular nanoplates using nonlocal third-order shear deformation plate theory

Shahrokh Hosseini-Hashemi [a, b], Mehdi Kermajani [a], Reza Nazemnezhad [a, *]

[a] School of Mechanical Engineering, Iran University of Science and Technology, Narmak, 16842-13114, Tehran, Iran
[b] Center of Excellence in Railway Transportation, Iran University of Science and Technology, Narmak, 16842-13114, Tehran, Iran

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In this paper, analytical closed-form solutions in explicit forms are presented to investigate small scale effects on the buckling and the transverse vibration behavior of Lévy-type rectangular nanoplates based on the Reddy's nonlocal third-order shear deformation plate theory. Two other edges of Lévy-type rectangular nanoplates may be restrained by different combinations of free, simply supported, or clamped boundary conditions. Hamilton's principle is used to derive the nonlocal equations of motion and natural boundary conditions of the nanoplate. Two comparison studies with analytical and numerical techniques reported in literature are carried out to demonstrate the high accuracy of the present new formulation. Comprehensive benchmark results with considering the small scale effects on frequency ratios, buckling load ratios, non-dimensional fundamental natural frequencies and non-dimensional buckling loads of rectangular nanoplates with different combinations of boundary conditions are presented for various values of nonlocal parameters, aspect ratios and thickness to length ratios. It is observed that except for SFSF rectangular nanoplates, as the aspect ratio increases, buckling load and natural frequency decreases, while keeping all other parameters fixed. For SFSF rectangular nanoplates, by increasing the aspect ratio, the values of the buckling load and frequency ratio increase.

1. Introduction

Many micro and nano devices consist of beams and plates suspended horizontally over a substrate. On the micro scale, suspended plates serve as the active component of accelerometers, rate gyroscopes, pressure sensors, chemical sensors, switches, electrostatic actuators, valves, and pumps. It is reasonable to assume that suspended plates will play a similarly important role on the nano scale. Therefore, at the design stage, accurate determination of natural frequencies and buckling load of nanoplates is very crucial for designers and engineers. To this end, one must consider small scale effects in order to refine classical theories to derive the governing equations for nano size structures. The scale effects are accounted by considering internal size as a material parameter. Experimental results show that as length scales of a material are reduced, the influences of long-range interatomic and intermolecular cohesive forces on the mechanical properties become prominent and cannot be neglected.

For modeling nanomaterials, a superior theory called nonlocal theory has been introduced to account for both features of lattice parameter and classical elasticity. Nonlocal theory of Eringen (Eringen, 2002) is one of the well-known continuum mechanics theories to capture the small scale effect by specifying the stress at a reference point as a functional of the strain field at every point in the body. Hence, many papers dealt with analyzing nano-structures have been published on this topic (Bedroud et al., 2013; Hosseini-Hashemi et al., 2013b, 2013c; Lu et al., 2007; Nazemnezhad and Hosseini-Hashemi, 2014; Reddy, 2007). Besides, some researchers suggested definitions for nonlocal parameter and showed its effect on natural frequency and critical buckling load (Aghababaei and Reddy, 2009; Aksencer and Aydogdu, 2011, 2012; Ansari et al., 2011a; Farajpour et al., 2012; Hosseini-Hashemi et al., 2013a, 2014, 2013d; Murmu and Pradhan, 2009a, b; Pradhan, 2009; Pradhan and Phadikar, 2009; Wang et al., 2007). For example, Wang et al. (2007) investigated nano scale effect on the free vibration of Timoshenko beam via nonlocal elasticity theory; and Pradhan and Phadikar (2009) reported vibration of multilayered graphene sheets through nonlocal and Mindlin theories. In another work,

* Corresponding author. Permanent address: School of Engineering, Damghan University, Damghan, Iran. Tel.: +98 233 525 1923; fax: +98 217 724 0488.
E-mail addresses: rnazemnezhad@iust.ac.ir, rnazemnezhad@du.ac.ir (R. Nazemnezhad).

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Aghababaei and Reddy (2009) derived equations of motion for nonlocal third-order shear deformation plate theory and considered effects of the nonlocal parameter on natural frequencies of a simply supported rectangular nanoplate. Pradhan (2009) analyzed Buckling of single layer graphene sheet based on nonlocal elasticity and higher order shear deformation theory. Furthermore, Farajpour et al. (2012) studied buckling of orthotropic micro/nanoscale plates under linearly varying in-plane load using nonlocal continuum mechanics. It is seen from literature survey that the solution of the governing equation, however, is based on numerical methods (finite element method (Ansari et al., 2010a), finite difference method (Ansari et al., 2011b), Galerkin method (Shen et al., 2012), differential quadrature method (Ansari et al., 2010b; Pradhan and Kumar, 2010)) and approximate analytical methods, such as Navier type solution method which assumes the variation of displacement variables harmonically (Aghababaei and Reddy, 2009; Lu et al., 2007). In addition, many of these solutions are based on nonlocal classical and Mindlin theories with Navier boundary condition in which all edges are simply supported while in few cases, combinations of clamed and simply supported boundaries have been taken into account (Aksencer and Aydogdu, 2011; Pradhan and Kumar, 2011). Therefore, no analytical closed-form solution is available in the literature for the static and dynamic analysis of thicker nano scale plate-like structures, multi-layer graphene and graphite, with various boundary conditions. Since, at its simplest approximation, a multi-layer graphene sheet may be represented by an elastic plate with an equivalent thickness and Young's modulus like the work done by Chandra et al. (2011) for bilayer graphene sheets, it may be reasonable using higher order shear deformation plate theory to investigate static and dynamic behavior of multi-layer graphene sheets.

According to the works of Bedroud et al. (2013), Hosseini-Hashemi et al. (2013a, 2011a, 2011b, 2008, 2013d), an analytical closed form solution procedure has been established for vibration and buckling analyses of single-layered and functionally graded plates based on some auxiliary and potential functions. This method has only been considered for local theories as well as nonlocal Kirchhof and Mindlin theories, and can be applied to Levy-type support conditions and yield results without any approximations. Therefore, the main purpose of this article is to utilize this analytical method to solve the governing equations of thick nanoplate for Reddy plate theory based on nonlocal elasticity. In this paper analytical closed-form solutions in explicit forms will be presented for transverse vibration and buckling analyses of rectangular thick nanoplates, multi-layer graphene and graphite, based on the Reddy's nonlocal third-order shear deformation plate theory. Hamilton's principle will be used to derive the equations of motion and natural boundary conditions of the nanoplate. This study has ability to capture both small scale effects and quadratic variation of shear strain and consequently shear stress through the nanoplate thickness. Benchmark results for natural frequencies and critical buckling loads of rectangular nanoplates with different combinations of boundary conditions are tabulated for various values of nonlocal parameter, aspect ratios and thickness to length ratios.

2. Problem formulation

2.1. Review of nonlocal theory

As mentioned earlier, in the nonlocal theory, the stress in a material body point is a function of strain field of the same point and all other ones in material domain. Thus, the stress tensor plays the essential role in this continuum theory which is defined as (Eringen, 2002):

$$t_{ij} = \int_{v} \alpha(|x' - x|)\sigma_{ij}(x')\mathrm{d}v' \quad (1)$$

where the volume integral is taken over the body region v; x is the reference point in body which the stress tensor is calculated at any other point like x' in the body; $i, j = x, y, z$ for three dimensional Cartesian coordinate; σ_{ij} is the local stress tensor and $\alpha(|x' - x|)$ is nonlocal kernel function depends on internal characteristic length. Eringen proposed $\alpha(|x' - x|)$ as a Green function of a linear differential operator $\mathscr{L}$ as:

$$\mathscr{L}\alpha(|x' - x|) = \delta(|x' - x|) \quad (2)$$

Substituting Eq. (2) into Eq. (1), the integral form of nonlocal stress tensor reduces to the differential one as follows:

$$\mathscr{L}t_{ij} = \sigma_{ij} \quad (3)$$

The linear operator is an approximate model of the kernel obtained by matching the Fourier transforms of the kernel in the wave number space with the dispersion curves of lattice dynamics. For curve-fitting at low wave numbers relevant to the small internal length scale, Eq. (2) is written as:

$$\left(1 - \varepsilon^2\nabla^2 + \gamma^4\nabla^4 - \dots\right)t_{ij} = \sigma_{ij} \quad (4)$$

Thus, the linear operator becomes:

$$\mathscr{L} = \left(1 - \varepsilon^2\nabla^2 + \gamma^4\nabla^4 - \dots\right) \quad (5)$$

whereε and γ are small parameters proportional to the internal length scale. If first order approximation is to be considered, just the Laplacian form of the operator in Eq. (5) is maintained (Alavinasab, 2009). Therefore, for the two-dimensional case:

$$\mathscr{L} = 1 - (e_0 l)^2\nabla^2 \quad (6)$$

in which l is internal length and e_0 is material constant which is defined by the experiment and $\nabla^2 = (\partial^2/\partial x^2) + (\partial^2/\partial y^2)$ is the two-dimensional Laplacian operator.

Equations of motion for nonlocal linear elastic solids are obtained from nonlocal balance law as:

$$t_{ij,j} + f_i = \rho\ddot{u}_i \quad (7)$$

where f_i and u_i are the components of the body force and displacement vector, respectively, and ρ is the mass density. Substituting Eq. (3) into Eq. (7), the nonlocal equations of motion in a differential form can be expressed by:

$$\sigma_{ij,j} + \mathscr{L}\left(f_i - \ddot{u}_i\right) = 0 \quad (8)$$

It should be noted that the boundary conditions here are based on nonlocal stress tensors t_{ij} rather than local ones σ_{ij} (Lu et al., 2007).

2.2. Geometrical configuration

Consider a flat, isotropic, and thick rectangular nanoplate of length a, width b, and uniform thickness h, as shown in Fig. 1. It is evident that the Levy type plate has two opposite edges simply supported along x_2 axis (i.e. along the edges $x_1 = 0$ and $x_1 = a$) whereas the other two edges may be free, simply supported or