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Numerical assessment, implementation and application of an extended Gurson model accounting for void size effects

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ABSTRACT

Using recent results obtained by Dormieux and Kondo (2010) on strength properties of ductile nanoporous media, we aim at investigating the mechanical behaviour of this class of materials. To this end, we first assess and validate the yield criterion proposed by these authors by performing numerical limit-analyses, based on the standard finite element method including elasticity, of an elementary porous cell. This requires special care in order to account for surface stresses at the void's boundary. Then, taking advantage of the normality property for the flow rule to derive the porosity and void size evolution equations, we formulate a complete model for ductile nanoporous materials. Owing to the parametric form of the yield locus, a specific procedure is developed for the numerical implementation of the model. The problem of plastic correction of the elastic stress predictor is reduced to the resolution of two nonlinear equations. Finally, the model is applied to proportional loadings, for different stress triaxialities. The void size appears to have a strong influence on the mechanical behaviour and on the porosity evolution.

1. Introduction

Ductile fracture of metals occurs through growth and coalescence of voids. In order to describe the macroscopic behaviour of plastic porous materials, Gurson (1977) derived a model by combining homogenization and limit-analysis of a spherical (or cylindrical) cell made of a von Mises matrix containing a spherical (or cylindrical) void, and subjected to conditions of homogeneous boundary strain rate. Gurson (1977)'s model has been extended in several directions, including the consideration of void shape effects, by Gologanu et al. (1993, 1994, 1997); Monchiet et al. (2014) for spheroidal cavities, and more recently Madou and Leblond (2012a,b) for arbitrary ellipsoidal voids. Other extensions have been proposed in order to account for matrix plastic anisotropy, by Benzerga and Besson (2001) for spherical or cylindrical voids, and Monchiet et al. (2008); Keralavarma and Benzerga (2010) for spheroidal voids.

Despite their great interest in practical situations, none of the above models is suitable for the description of the growth of submicron voids. Indeed, it is well known that the plastic flow strength and the hardening of a solid depend on the gradient of the plastic strain; the effect is important for plastic deformations taking place at small scales, of the order of a micron or less, which is the case when submicron voids are present. To account for such effects, Fleck and Hutchinson (1993), Fleck et al. (1994), Fleck and Hutchinson (1997), developed a phenomenological model of plasticity known as strain gradient plasticity, whereby the material behaviour depends on both the plastic strain and the plastic strain gradient. This model has permitted to evidence size effects in void growth, first for an isolated void (Huang et al., 2000; Fleck and Hutchinson, 2001) and then for finite porosities (Niordson, 2008). In addition to these numerical works, Monchiet and Bonnet (2013) revisited Gurson (1977)'s homogenization of a hollow sphere, by considering a matrix obeying a strain gradient plasticity model; their yield criterion, obtained by introducing some approximations to calculate the macroscopic plastic dissipation, exhibited a void size dependency: when the void size decreases, the size of the yield locus increases and the void growth rate decreases.

If size effects due to strain gradients are well understood for micron-sized voids, the physics of nano-sized voids, in contrast, is still in its infancy. Various atomistic simulations have evidenced a drastic increase of the strength of nanovoids (Traiviratana et al., 2008; Mi et al., 2011; Tang et al., 2014), due to the presence of surface effects at the interface between the bulk material and the

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empty void; these effects are commonly attributed to surface stresses and nucleation of dislocations. In order to account for them Dormieux and Kondo (2010) extended Gurson (1977)'s model by representing the cavity surface through a stress interface model (Gurtin and Murdoch, 1975, 1978), that is by schematizing it as a thin layer obeying a plane stress von Mises (isotropic) criterion.[1] This interface model consists in a simple heuristic continuum mechanics representation of crystalline plasticity that takes place at the nanoscale; even though this kind of criterion may be questionable at such a scale, it is acceptable as a first approximation and permits to perform analytically a limit-analysis of a hollow sphere including surface effects. The macroscopic yield criterion obtained by Dormieux and Kondo (2010) is expressed in terms of parametric equations of the macroscopic yield locus, and exhibits a void size dependency: again, the size of the yield locus increases when the pore size decreases.

The assessment of Dormieux and Kondo (2010)'s macroscopic yield criterion, and the numerical integration of the corresponding constitutive equations, which are needed for future applications of the model to the growth of nanosized voids, are the main objectives of this paper.

The paper is organized as follows:

- In Section 2, we numerically assess Dormieux and Kondo (2010)'s criterion by performing numerical limit-analyses of a hollow sphere with an interface model at the void's boundary, based on the standard finite element method including elasticity.
- The criterion is then completed in Section 3 by providing the flow rule and the evolution equations of the internal parameters.
- In Section 4, a numerical implementation of the complete model is proposed. The local algorithm of plastic correction of the elastic stress predictor is reduced to the resolution of two coupled nonlinear equations on two unknowns.
- Finally, Section 5 provides a numerical application of the model, in the form of computation of the behaviour of a homogeneous volume element subjected to some axisymmetric proportional loading at fixed stress triaxiality.

2. Numerical assessment of Dormieux and Kondo (2010)'s criterion

2.1. Dormieux and Kondo (2010)'s macroscopic criterion

We consider Dormieux and Kondo (2010)'s homogenized criterion for ductile nanoporous materials, which has been obtained from a limit-analysis of a single hollow sphere (see Fig. 1).

The strength of the solid phase is described by a von Mises criterion:

$$\sigma_{eq}^2 = \frac{3}{2}\boldsymbol{\sigma}_d : \boldsymbol{\sigma}_d \leq \sigma_0^2 \tag{1}$$

where σ_{eq} is the von Mises equivalent stress, $\boldsymbol{\sigma}_d$ the deviator of the stress tensor $\boldsymbol{\sigma}$ and σ_0 the yield stress in simple tension. The void size effect is introduced by accounting for the presence of an interface at the void's boundary. The strength of this interface is described by a 2D plane stress von Mises-type criterion:

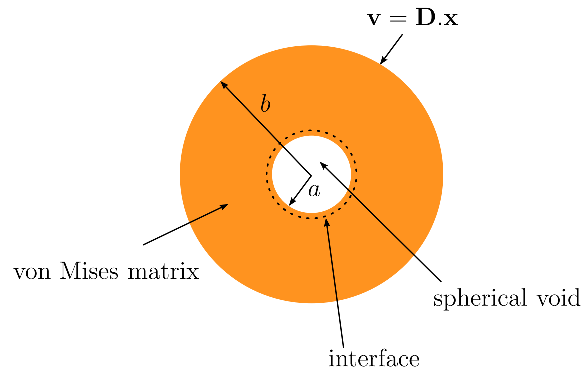

Fig. 1. Representative Volume Element for the nanoporous material.

$$\frac{3}{2}\boldsymbol{\sigma}_d^S : \boldsymbol{\sigma}_d^S \leq k_{int}^2 \tag{2}$$

where $\boldsymbol{\sigma}_d^S$ is the deviatoric part of the plane stress tensor $\boldsymbol{\sigma}^S$ and k_{int} represents the yield limit of the interface. (Note that the plane stress tensor $\boldsymbol{\sigma}^S$ is expressed in N/m, not in N/m^2). The void size effect is thus characterized by a non-dimensional parameter Γ, depending on the void radius a, the interface strength k_{int} and the yield stress σ_0, defined as:

$$\Gamma = \frac{k_{int}}{a\sigma_0}. \tag{3}$$

In the following, Γ will be referred to as the "interface non-dimensional parameter".

Both the bulk material and the interface obey the plastic flow rule associated to the relevant yield criterion through normality.

The limit-analysis of the hollow sphere of internal radius a, external radius b, porosity $f = a^3/b^3$, taking account of a von Mises interface, leads to the following parametric equations of the macroscopic yield locus (Dormieux and Kondo, 2010):

$$\begin{cases} \mathrm{tr}\,\boldsymbol{\Sigma} = \sigma_0 \left[2\Big(\mathrm{Argsinh}(\xi) - \mathrm{Argsinh}(f\xi) \Big) + \Gamma \dfrac{6\xi}{\sqrt{\xi^2 + 3/5}} \right] \\ \Sigma_{eq} = \sigma_0 \left[\sqrt{1 + f^2\xi^2} - f\sqrt{1+\xi^2} + \Gamma \dfrac{9f}{5\sqrt{\xi^2 + 3/5}} \right] \end{cases} \tag{4}$$

where $\boldsymbol{\Sigma}$ denotes the macroscopic stress tensor, Σ_{eq} the macroscopic von Mises equivalent stress, and the parameter $\xi = (2/f)(D_m/D_{eq})$ is defined from the mean and equivalent deviatoric parts D_m, D_{eq} of the macroscopic strain rate $\mathbf{D}$.

2.2. Numerical limit-analysis

It has been shown by Madou and Leblond (2012b) that limit-analysis problems can be solved by the standard finite element method including elasticity, by using a large load increment without geometry update. Indeed, under such conditions the time-discretized equations of the finite element problem become equivalent to those of limit-analysis, provided that the local problem of "projection onto the yield locus" (plastic correction of the elastic stress predictor) is solved using an implicit algorithm.

We use the commercial finite element code SYSTUS® developed by ESI Group to perform the calculations. We consider a spherical

[1] Recently, Monchiet and Kondo (2013) extended Dormieux and Kondo (2010)'s work by considering Gologanu et al. (1993, 1994)'s criterion for spheroidal voids instead of that of Gurson (1977) for spherical ones.