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Configurational forces for quasi-incompressible large strain electro-viscoelasticity – Application to fracture mechanics

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ABSTRACT

This work treats theoretical and numerical aspects of configurational forces with application to fracture mechanics of electroactive polymers (EAPs) modelled in the continuum mechanics based context of large strain quasi-incompressible electro-viscoelasticity. We adopt a four-field formulation to take the quasi-incompressible behaviour in a finite element framework into account. We discuss the numerical implementation of viscosity-related evolution equations and the implications of viscous internal variables on the computation of configurational forces. As numerical examples for time depending crack driving forces we discuss a pre-cracked stacked EAP actuator as well as a pre-cracked and pre-stretched block actuator.

1. Introduction

Electroactive polymers (EAP) are an interesting class of materials that show electro-mechanical coupling. An applied electric field leads to a mechanical deformation of the EAP, and thus this class of materials is of continuously growing interest in technical applications such as actuators, sensors, micro-robotics, biomimetrics and energy harvesting, see e.g. Kornbluh et al. (1998), Pelrine et al. (2000), Bar-Cohen Stewart et al. (2002), Samatham et al. (2007), Capri et al. (2008). In all these applications EAPs undergo cyclic deformation, thus fracture mechanics in the sense of fatigue crack growth are of key importance. In this regard, the theory of configurational forces provides a suitable theory to calculate crack driving forces, see Maugin (1993), Gurtin (2000), Trimarco and Maugin (2001), Maugin (2011). Related to the numerical modelling of EAPs, finite element implementations were reported, e.g. by Vu et al. (2007), Vu and Steinmann (2007b), Gao et al. (2011), Klassen et al. (2012), Henann et al. (2013), Klinkel et al. (2013) for the electro-elastostatic case and by Park et al. (2012) for the dynamical electro-elastic case. For large strain electro-elastostatics, Vu and Steinmann (2007b) presented the numerical computation of discrete configurational forces for a pre-cracked specimen under Mode-I load as well, whereas Miehe et al. (2010) and Xu et al. (2010b) developed electromechanical fracture models based on a phase field theory for piezoelectric materials. A further branch of current research deals with the enhancement of the electro-mechanical coupling by microstructural optimisation, see e.g. Tian et al. (2012), Rudykh et al. (2013), Gei et al. (2013). Most works are based on the modelling of electro-mechanical behaviour of EAPs by electro-hyperelasticity, whereas the pronounced visco-elastic behaviour of EAPs was very recently introduced in the context of the finite element method by Ask et al. (2012a,b) and Bueschel et al. (2013).

Our work focuses on the theoretical and numerical aspects of configurational forces with application to EAPs modelled in the continuum mechanics based context of large strain quasi-incompressible electro-viscoelasticity. The paper is organised as follows: In Sections 2–4 we discuss kinematics, electric field, balance laws and a general constitutive model for electro-viscoelasticity. Section 5 deals with the introduction of a four field formulation of the weak form and its discretisation to take the quasi-incompressible material behaviour of EAPs into account. The notion of configurational forces in the context of electro-viscoelasticity and their numerical computation is addressed in Section 6. Finally, we present the material model together with its numerical implementation and discuss two- and three-dimensional numerical examples for pre-cracked EAP actuators in Section 7.

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2. Kinematics and electric field

To introduce notation we briefly summarise the geometrically nonlinear kinematics of the spatial motion problem, see also Fig. 1. In the spatial motion problem the placement $\boldsymbol{x}$ of a 'physical particle' in the spatial configuration $\mathscr{B}_t$ at time t is described by a nonlinear deformation mapping $\boldsymbol{x} = \varphi(\boldsymbol{X},t)$ in terms of the placement $\boldsymbol{X}$ of the same 'physical particle' in the chosen material configuration $\mathscr{B}_0$ at time $t_0 = 0$. The deformation gradient, i.e. the linear tangent map associated with the deformation mapping, is given by $\boldsymbol{F} = \nabla_{\boldsymbol{X}}\varphi$ together with its determinant $J = \det \boldsymbol{F}$. As this work proceeds, we assume purely isochoric behaviour of the viscous part of our electro-viscoelastic material model and, moreover, we split the deformation gradient multiplicatively into its volumetric part J and its isochoric part $\overline{\boldsymbol{F}} = J^{-1/3}\boldsymbol{F}$ with $\det \overline{\boldsymbol{F}} = 1$.

Furthermore, we use the constitutive framework of a multiplicative split of the isochoric part of the deformation gradient into an elastic and a viscous part. This can be interpreted as a generalisation of a one-dimensional rheological Maxwell element to three dimensions, see e.g. Lubliner (1990), Maugin (1992). When considering n parallel Maxwell elements, the multiplicative split of the isochoric part of the deformation gradient is introduced for each Maxwell element by

$$\overline{\boldsymbol{F}} = J^{-1/3}\boldsymbol{F} = \overline{\boldsymbol{F}}_{e\alpha}\cdot\boldsymbol{F}_{v\alpha} \quad \text{with } \alpha = 1,\ldots,n \tag{1}$$

In addition, we assume that the viscous part $\boldsymbol{F}_{v\alpha}$ is purely isochoric, i.e. $\det \boldsymbol{F}_{v\alpha} = 1$, thus $\overline{\boldsymbol{F}}_{e\alpha} = J^{-1/3}\boldsymbol{F}_{e\alpha}$ with $\boldsymbol{F}_{e\alpha} = \boldsymbol{F}\cdot\boldsymbol{F}_{v\alpha}^{-1}$ and $\det \overline{\boldsymbol{F}}_{e\alpha} = 1$. With this at hand, we adopt different right Cauchy-–Green-type tensors as deformation measures referring to

$$\boldsymbol{C} = \boldsymbol{F}^{t}\cdot\boldsymbol{F} \quad \text{total deformation} \tag{2}$$

$$\overline{\boldsymbol{C}} = \overline{\boldsymbol{F}}^{t}\cdot\overline{\boldsymbol{F}} \quad \text{total isochoric deformation} \tag{3}$$

$$\overline{\boldsymbol{C}}_{e\alpha} = \overline{\boldsymbol{F}}^{t}_{e\alpha}\cdot\overline{\boldsymbol{F}}_{e\alpha} \quad \text{elastic isochoric deformation} \tag{4}$$

$$\boldsymbol{C}_{v\alpha} = \boldsymbol{F}^{t}_{v\alpha}\cdot\boldsymbol{F}_{v\alpha} \quad \text{viscous (isochoric) deformation} \tag{5}$$

Assuming that no magnetic fields and no free currents are present, the electric field vector satisfies Faraday's law in the form

$$\nabla_{\boldsymbol{X}}\times\boldsymbol{E} = 0 \ \text{ in } \mathscr{B}_0 \ \text{ and } \ \nabla_{\boldsymbol{x}}\times\boldsymbol{e} = 0 \ \text{ in } \mathscr{B}_t \tag{6}$$

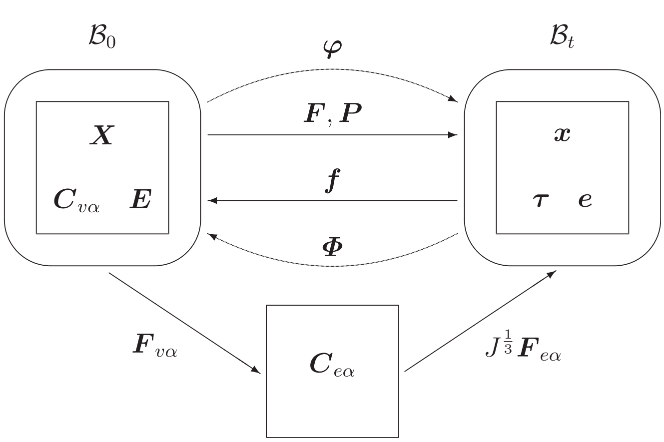

Fig. 1. Fields in the material and spatial configuration.

Thus, the electric field vector can be expressed as the gradient of an electric potential ϕ as

$$\boldsymbol{E} = -\nabla_{\boldsymbol{X}}\phi \ \text{ in } \mathscr{B}_0 \ \text{ and } \ \boldsymbol{e} = -\nabla_{\boldsymbol{x}}\phi \ \text{ in } \mathscr{B}_t \tag{7}$$

whereby the material electric field vector $\boldsymbol{E} = \boldsymbol{e}\cdot\boldsymbol{F}$ is seen as the pull back of the spatial electric field vector $\boldsymbol{e}$.

3. Balance laws

The quasi-static balance of momentum of the spatial motion problem in local form and in the presence of an electric field is given by

$$\nabla_{\boldsymbol{x}}\cdot\boldsymbol{\tau} + \boldsymbol{b}_t = 0 \quad \text{in } \mathscr{B}_t \tag{8}$$

where $\boldsymbol{b}_t$ is the volume specific body force. The total spatial stress, see Maugin (1988), Hutter et al. (2006), Dorfmann and Ogden (2006), is introduced as

$$\boldsymbol{\tau} = \boldsymbol{\sigma} + \boldsymbol{e}\otimes\boldsymbol{d} - \frac{1}{2}\varepsilon_0[\boldsymbol{e}\cdot\boldsymbol{e}]\boldsymbol{I} \tag{9}$$

Hereby $\boldsymbol{\sigma}$ is the Cauchy stress, $\boldsymbol{d}$ the spatial electric displacement, both of which have to be determined by constitutive relations, $\boldsymbol{I}$ represents the second order identity tensor and ε_0 denotes the permittivity in vacuum. The corresponding local balance of momentum referring to the material configuration $\mathscr{B}_0$ is based on the total Piola-type stress tensor $\boldsymbol{P} = \boldsymbol{\tau}\cdot\operatorname{cof}\boldsymbol{F}$, with the cofactor $\operatorname{cof}\boldsymbol{F} = J\boldsymbol{F}^{-t}$, and reads

$$\nabla_{\boldsymbol{X}}\cdot\boldsymbol{P} + \boldsymbol{b}_0 = 0 \ \text{ and } \ \boldsymbol{b}_0 = J\boldsymbol{b}_t \ \text{ in } \mathscr{B}_0 \tag{10}$$

as $\nabla_{\boldsymbol{X}}\cdot\operatorname{cof}\boldsymbol{F} = 0$. The balance law with respect to the electric field for the special case of vanishing electric charges is the electric Gauss' law

$$\nabla_{\boldsymbol{x}}\cdot\boldsymbol{d} = 0 \quad \text{in } \mathscr{B}_t \tag{11}$$

with the spatial electric displacement $\boldsymbol{d}$.

In the material configuration $\mathscr{B}_0$ the corresponding electric displacement is given as $\boldsymbol{D} = \boldsymbol{d}\cdot\operatorname{cof}\boldsymbol{F}$, and thus the balance law can also be expressed as

$$\nabla_{\boldsymbol{X}}\cdot\boldsymbol{D} = 0 \quad \text{in } \mathscr{B}_0 \tag{12}$$

4. Constitutive relations

As this work proceeds, we assume an energy function of the form

$$\Omega_0(\boldsymbol{F},\boldsymbol{E},\boldsymbol{C}_{v\alpha}) = \psi_0^{vol}(J) + \psi_0^{\infty}(\overline{\boldsymbol{F}}) + \sum_\alpha \psi_0^{\alpha}(\overline{\boldsymbol{F}},\boldsymbol{C}_{v\alpha}) + \psi_0^{mel}(\overline{\boldsymbol{F}},\boldsymbol{E}) + \psi_0^{el}(\boldsymbol{E}) - \frac{1}{2}\varepsilon_0 J\boldsymbol{C}^{-1}:[\boldsymbol{E}\otimes\boldsymbol{E}] \tag{13}$$

where $\psi_0^{vol}(J)$ and $\psi_0^{\infty}(\overline{\boldsymbol{F}})$ describe the volumetric and isochoric hyperelastic response, and where $\sum_\alpha \psi_0^{\alpha}(\overline{\boldsymbol{F}},\boldsymbol{C}_{v\alpha})$ describes the viscous behaviour of the material. The electro-mechanical coupling is modelled by $\psi_0^{mel}(\overline{\boldsymbol{F}},\boldsymbol{E})$, and the contribution $\psi_0^{el}(\boldsymbol{E}) - 1/2\varepsilon_0 J\boldsymbol{C}^{-1}:[\boldsymbol{E}\otimes\boldsymbol{E}]$ defines the purely electrostatic behaviour; see, e.g., Vu and