European Journal of Mechanics A/Solids 45 (2014) 211–225

Contents lists available at ScienceDirect

European Journal of Mechanics A/Solids

journal homepage: www.elsevier.com/locate/ejmsol

Analysis of functionally graded sandwich plates using a new first-order shear deformation theory

Huu-Tai Thai [a,*], Trung-Kien Nguyen [b], Thuc P. Vo [c], Jaehong Lee [d]

[a] Centre for Infrastructure Engineering and Safety, School of Civil and Environmental Engineering, The University of New South Wales, Sydney, NSW 2052, Australia

[b] Faculty of Civil Engineering and Applied Mechanics, University of Technical Education Ho Chi Minh City, 1 Vo Van Ngan Street, Thu Duc District, Ho Chi Minh City, Vietnam

[c] Faculty of Engineering and Environment, Northumbria University, Newcastle upon Tyne NE1 8ST, UK

[d] Department of Architectural Engineering, Sejong University, 98 Kunja Dong, Kwangjin Ku, Seoul 143-747, Korea

A R T I C L E I N F O

Article history:
Received 8 August 2013
Accepted 17 December 2013
Available online 25 December 2013

Keywords:
Functionally graded sandwich plate
Plate theory
Bending
Buckling
Free vibration

A B S T R A C T

In this paper, a new first-order shear deformation theory is presented for functionally graded sandwich plates composed of functionally graded face sheets and an isotropic homogeneous core. By making a further assumption to the existing first-order shear deformation theory, the number of unknowns and governing equations of the present theory is reduced, thereby making it simple to use. In addition, the use of shear correction factor is no longer necessary in the present theory since the transverse shear stresses are directly computed from the transverse shear forces by using equilibrium equations. Equations of motion are derived from Hamilton's principle. Analytical solutions for bending, buckling and free vibration analysis of rectangular plates under various boundary conditions are presented. Verification studies show that the present first-order shear deformation theory is not only more accurate than the conventional one, but also comparable with higher-order shear deformation theories which have a greater number of unknowns.

1. Introduction

Functionally graded materials (FGMs) are a class of composites that have continuous variation of material properties from one surface to another, and thus eliminating the stress concentration found in laminated composites. A typical FGM is made from a mixture of ceramic and metal. These materials are often isotropic but nonhomogeneous. The reason for interest in FGMs is that it may be possible to create certain types of FGM structures capable of adapting to operating conditions.

Sandwich structures composed of a core bonded to two face sheets are commonly used in the aerospace vehicles due to their outstanding bending rigidity, low specific weight, excellent vibration characteristics and good fatigue properties. However, the sudden change in the material properties from one layer to another can result in stress concentrations which often lead to delamination. To overcome this problem, the concept of functionally graded (FG) sandwich structures is proposed. In such materials, two face sheets are made from isotropic FGMs while the core is made from an isotropic homogeneous material. Thanks to the smooth and continuous variation in the properties of FGMs, the stress concentration which is found in laminated sandwich structures is eliminated in FG sandwich structures.

With the wide application of FG sandwich structures, understanding their responses becomes an essential task. Since the shear deformation effect is more pronounced in thick plates or plates made of advanced composites like FGM, shear deformation theories such as first-order shear deformation theory (FSDT) and higher-order shear deformation theories (HSDT) should be used to predict the responses of FG sandwich plates. The FSDT gives acceptable results but depends on the shear correction factor which is hard to determine since it depends on many parameters. Conversely, the HSDT do not require shear correction factor, but its equations of motion are more complicated than those of the FSDT. It is observed from the literature that most studies on FG sandwich plates are based on HSDTs. Zenkour (2005a,b) used a sinusoidal shear deformation theory (SSDT) to study the bending, buckling and free vibration of FG sandwich plates with FG face sheets and a homogeneous core. The behavior of FG sandwich plates under thermal environment was also studied by Zenkour and Alghamdi (2008, 2010) and Zenkour and Sobhy (2010) using FSDT, SSDT and third-order shear deformation theory (TSDT) of Reddy (1984). Based on an accurate HSDT, Natarajan and Manickam (2012)

* Corresponding author. Tel.: +61 2 9385 5029.
E-mail address: t.thai@unsw.edu.au (H.-T. Thai).

0997-7538/$ – see front matter © 2013 Elsevier Masson SAS. All rights reserved.
http://dx.doi.org/10.1016/j.euromechsol.2013.12.008

studied the bending and free vibration behavior of two types of FG sandwich plates, i.e. homogeneous face sheets with a FG core and FG face sheets with a homogenous core. Neves et al. (2012a,b, 2013) developed HSDT to predict the behavior of FG sandwich plates. Recently, Xiang et al., (2013) analyzed the free vibration of FG sandwich plates using a nth-order shear deformation theory and a meshless method, while Sobhy (2013) investigated the buckling and free vibration of FG sandwich plates using various HSDTs.

It should be noted that HSDTs are highly computational cost due to involving in many unknowns (e.g., theories (Neves et al., 2012b, 2013) with nine unknowns and (Natarajan and Manickam, 2012) with thirteen unknowns). To reduce computational cost, HSDTs with four unknowns were recently developed for FG sandwich plates (see references (Abdelaziz et al., 2011; Ahmed Houari et al., 2011; El Meiche et al., 2011; Hadji et al., 2011; Merdaci et al., 2011; Bourada et al., 2012; Hamidi et al., 2013; Tounsi et al., 2013)). Although the existing FSDT is widely used to develop finite element models due to its simplicity, its accuracy is strongly dependent on the proper value of the shear correction factor. As a result, it is inconvenient to use. In this paper, a new FSDT which eliminates the use of the shear correction factor is developed for FG sandwich plates composed of FG face sheets and an isotropic homogeneous core. By making a further assumption, the number of unknowns and governing equations of the present FSDT is reduced, thus makes it simple to use. Equations of motion and boundary conditions are derived from Hamilton's principle. Analytical solutions for rectangular plates under various boundary conditions are obtained. Numerical examples are presented to verify the accuracy of the present theory in predicting the bending, buckling and free vibration responses of FG sandwich plates.

2. Theoretical formulation

Consider a sandwich plate composed of three layers as shown in Fig. 1. Two FG face sheets are made from a mixture of a metal and a ceramic, while a core is made of an isotropic homogeneous material. The material properties of FG face sheets are assumed to vary continuously through the plate thickness by a power law distribution as

$$P(z) = P_m + (P_c - P_m)V \tag{1}$$

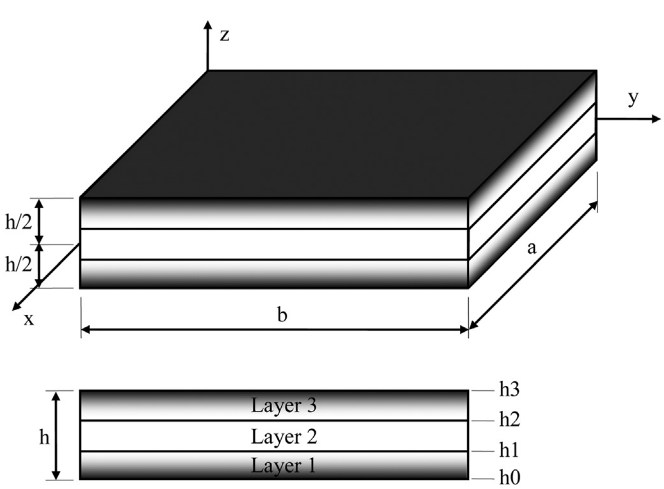

Fig. 1. Geometry and coordinates of FG sandwich plates.

where P represents the effective material property such as Young's modulus E, Poisson's ratio ν, and mass density ρ; subscripts c and m denote the ceramic and metal phases, respectively; and V is the volume fraction of the ceramic phase defined by

$$\begin{cases} V^{(1)}(z) = \left(\frac{z-h_0}{h_1-h_0}\right)^p & \text{for } z\in[h_0,h_1] \\ V^{(2)}(z) = 1 & \text{for } z\in[h_1,h_2] \\ V^{(3)}(z) = \left(\frac{z-h_3}{h_2-h_3}\right)^p & \text{for } z\in[h_2,h_3] \end{cases} \tag{2}$$

where p is the power law index that governs the volume fraction gradation. Fig. 2 shows the through thickness variation of the volume fraction of the ceramic phase for five different schemes considered in this study.

2.1. Kinematics

The displacement field of the conventional FSDT is given by

$$\begin{aligned} u_1(x,y,z) &= u(x,y) + z\varphi_x(x,y) \\ u_2(x,y,z) &= v(x,y) + z\varphi_y(x,y) \\ u_3(x,y,z) &= w(x,y) \end{aligned} \tag{3}$$

where u, v, w, φ_x and φ_y are five unknown displacement functions of the midplane of the plate. By assuming $\varphi_x = -\partial\theta/\partial x$ and $\varphi_y = -\partial\theta/\partial y$, the displacement field of the new FSDT can be rewritten in a simpler form as

$$\begin{aligned} u_1(x,y,z) &= u(x,y) - z\frac{\partial\theta}{\partial x} \\ u_2(x,y,z) &= v(x,y) - z\frac{\partial\theta}{\partial y} \\ u_3(x,y,z) &= w(x,y) \end{aligned} \tag{4}$$

It is clear that the displacement field in Eq. (4) contains only four unknowns (u, v, θ, w). It is worth noting that the simple FSDT recently proposed by Thai and Choi (2013a,b) also involves only four unknowns like the present theory. However, the displacement field of the simple FSDT (Thai and Choi, 2013a,b) is obtained by splitting the transverse displacement into bending and shear parts instead of using a further assumption as in the present work. Therefore, the displacement field and subsequent equations of motion derived in this study will be completely different with those given by Thai and Choi (2013a,b). In addition, the present FSDT does not require a shear correction factor as in the case of the simple FSDT (Thai and Choi, 2013a,b).

The strains associated with the displacement field in Eq. (4) are

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} = \begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{Bmatrix} + z\begin{Bmatrix} \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{Bmatrix} \tag{5a}$$

$$\begin{Bmatrix} \gamma_{xz} \\ \gamma_{yz} \end{Bmatrix} = \begin{Bmatrix} \gamma_{xz}^0 \\ \gamma_{yz}^0 \end{Bmatrix} \tag{5b}$$

Eq. (5) can be rewritten in a compact form as

$$\{\varepsilon\} = \left\{\varepsilon^0\right\} + z\{\kappa\} \tag{6a}$$

$$\{\gamma\} = \left\{\gamma^0\right\} \tag{6b}$$