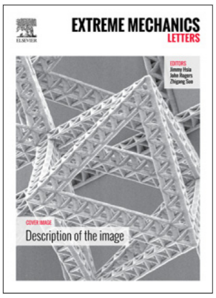

Extreme mechanics in laminated shells: New insights

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Extreme mechanics in laminated shells: new insights

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Abstract

Fibre-reinforced composite laminates are increasingly popular for lightweight applications in the aerospace, wind turbine and automotive industries. Apart from their excellent specific strength and stiffness properties, their orthotropic and layered construction can be exploited to create bistable systems that are useful for shape-adaptive morphing or energy-harvesting applications. The nonlinear interaction between initial curvature and induced thermal stresses can lead to "extreme" mechanics with rich branching and multi-stable behaviour. Here, we explore the multi-stability of curved composite laminates in the two-dimensional parameter space of temperature and mechanical loading. Analyses conducted via finite element discretisation and numerical continuation algorithms reveal possible snapping behaviour between five different mode shapes for different combinations of applied load and temperature. Furthermore, we show that a composite laminate can exhibit four stable, self-equilibrated mode shapes for a narrow temperature range. Hence, the multi-stable and nonlinear mechanics of composite laminates is much more complex than previously assumed, and this may influence the design of more versatile morphing structures.

Keywords: Nonlinear structures, Bifurcations, Morphing, Composite material

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1. Introduction

In structural engineering, elastic instabilities are historically viewed as a "failure" mechanism. An alternative perspective has developed over the last decade, whereby instabilities are used for additional functionality [1]. Example applications include energy harvesting [2, 3], reversible shape-adaptation [4, 5], surface texturing [6], actuation [7], self-encapsulation [8], auxetic materials [9] and energy dissipation [10].

In many of these applications, the underlying mechanical principle is multi-stability. Multi-stability often arises in geometrically curved structures in a stress-free state—*e.g.* a circular arch snapped between two inverted shapes—or alternatively, in curved geometries obtained from an induced stress field—*e.g.* the post-buckled state of the slender elastica. In the latter case, geometry and pre-stress can interact nonlinearly to produce very rich and oftentimes complex bifurcation behaviour, a classical example being the post-buckling response of the axially compressed cylinder [11]. Although this rich bifurcation behaviour can broaden the spectrum of potentially useful functionality, there are still open research questions to how this behaviour is best explored numerically [12], validated experimentally [13], and reliably controlled in practice [14].

Modern composite materials, such as carbon-fibre and fibre-glass laminates, are a natural source of complex bifurcation behaviour and interesting "extreme" mechanics. Composite laminates are often manufactured by stacking plies of fibre-reinforced plastic on a mould surface in a specific sequence. Although reinforced plastics with curvlinear fibres exist [15], traditionally the reinforcing fibres are straight and point in a specific direction, θ, with respect to a defined material axis. In this manner, a composite laminate can be assembled from individual composite layers with different fibre orientations, and the assembly then cured under elevated temperature and pressure. As long as the resin that binds fibres together is free to flow during cure, it can be assumed that the cured composite takes a stress-free condition at its elevated curing temperature. Upon cooling to room temperature, which is typically a ΔT in the region of $[-200, -160]\,\mathrm{K}$, thermally induced shrinkage is induced. Due to the orthotropy of fibre-reinforced plastics, each layer will want to contract/expand by different amounts and in different directions. However, the restraint enforced by polymer bonding between layers induces intralaminar tensile/compressive stresses. If the lamination sequence is symmetric—hence, for every θ-fibre orientation below the mid-surface of the laminate there exists a symmetric θ-layer above the laminate mid-surface—then the bending moments induced by these thermal stresses will balance about the mid-surface, and the laminate will retain the stress-free curvature. If the lamination sequence is non-symmetric, then a thermal bending moment is induced that warps the laminate. In this sense, a non-symmetric composite laminate is a two-dimensional extension of the ubiquitous bimetallic strip shown in Fig. 1.

As a non-symmetric composite laminate cools to room temperature, *i.e.* upon warping, it may "morph" onto dif-

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