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Geometric role in designing pneumatically actuated pattern-transforming metamaterials

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ABSTRACT

When a metamaterial, composed of an elastomer with periodic circular holes sealed by elastomeric membranes, is subject to a compressive load, it can undergo a pattern transformation, yielding a large transformation strain. Such pattern transformation can be broadly tuned by changing the geometric parameters of the metamaterial. We numerically, analytically and experimentally survey the design space of the geometric parameters, and investigate their effects on the pattern transformation. Our finite element simulations reveal that the slenderest wall thickness and the pattern of the holes play key roles in determining the critical load for the pattern transformation, the transformation strain, and the transformation type. To quantify the effects of these geometric parameters, we further analytically model the pattern transformation of the metamaterial by simplifying it to a network of rigid rectangles linked by deformable beams. Finally, we experimentally characterize the pattern transformation of the metamaterials with different geometric parameters. The experimental, numerical, and analytical results are in good agreement with each other. Our work provides design guidelines for this metamaterial.

1. Introduction

Mechanical metamaterials are materials with micro-architectures, which bring in unusual mechanical properties that are difficult to achieve in conventional materials [1,2]. Mechanical metamaterials with micro-/nano-lattice structures have reached unprecedentedly high stiffness and strength at an extremely low density [3,4]. Pentamode lattices have shown vanishing shear moduli [5–7]. Negative Poisson's ratio is found in auxetic metamaterials with various re-entrant structures [8–16]. In additional to the elastic constants, the whole nonlinear stress–strain relations of metamaterials are open to design. Introducing bistable structures into the micro-architectures enables pattern transformations of the metamaterials and non-monotonic stress–strain curves [17–19]. Making use of chiral lattice structures, a metamaterial can twist under a uniaxial compression [20].

Recently, a new class of mechanical metamaterials, each consisting of a rubber slab with periodic holes, has emerged [11,21–30]. These metamaterials can undergo a pattern transformation when subject to a compressive load. By harnessing the geometric nonlinearity and the pattern transformation of the metamaterials, researchers have shown a variety of unique functions of them, such as negative Poisson's ratio [11], tunable acoustic properties [30,31] and inducing torsional motions [29]. When the holes are alternatingly large and small, the stress–strain curves of the metamaterials change the monotonicity under different transverse constraints [24,26]. More interestingly, since the large and small holes collapse in two perpendicular directions, a high-symmetry square lattice of holes can transform into a low-symmetry rectangular lattice, with two energetically equivalent variants [28]. This phenomenon is analogue to the phase transformation in shape memory alloys. Indeed, the shape memory effort has been demonstrated in this kind of metamaterials [28].

The advantage of metamaterials is that their properties can be tuned by their micro-architectures [11,21,22,24–26,28]. In the metamaterials with periodic holes, the shape of the holes has been shown to tune the stiffness, Poisson's ratio, and critical condition for the pattern transformation [11,25]. The sizes of the large and small holes determine the transition between the monotonic and non-monotonic stress–strain responses [24,26]. To understand the effect of geometry on the critical condition for the pattern transformation, finite element simulations [11,21,22,25–28,32,33] and analytical models [32,33] have been developed for the metamaterials with uniform-sized holes. These analytical models can predict the critical condition for the pattern transformation based on buckling analysis of the slender ligaments. However, they fail to predict the transformation strains due to lack of post-buckling analysis. For the metamaterials with alternatingly large and small holes, an analytical model has been proposed to understand the programmable monotonicity of the stress–strain curves [24]. However, this model is simple and unable to capture the geometric effects.

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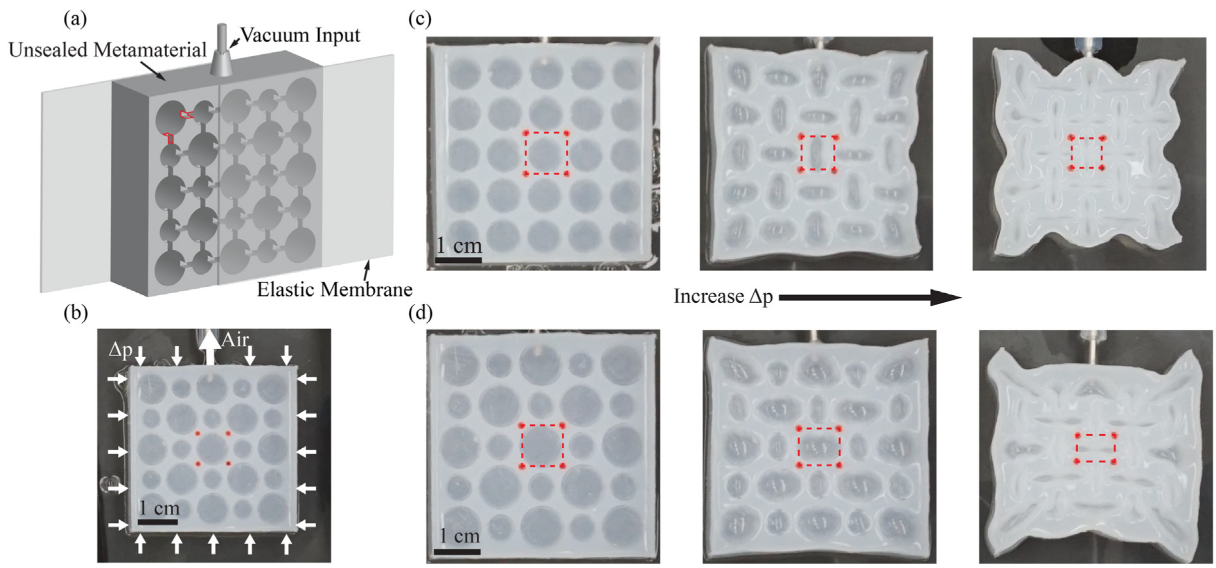

Fig. 1. Overview of the pneumatically actuated pattern-transforming metamaterials. (a) A schematic of the fabrication. The metamaterial is an elastomeric slab containing an array of holes. All the holes are sealed by two elastic membranes, isolating the inner air from the atmosphere. The air chambers formed by the holes are connected via grooves (two of which are outlined in red). The metamaterial is actuated by a source of vacuum, such as a syringe or a pump. (b) When actuated, the metamaterial is subjected to an external and internal pressure difference and undergoes a pattern transformation. (c) When the holes are uniform-sized, the square lattice keeps square and the transformation strain is equibiaxial. (d) When the holes are alternatingly large and small, the square lattice transits to a rectangular shape and the transformation strain is non-equibiaxial.

In this paper, combining finite element analysis, experiments and analytical modeling, we aim to establish quantitative understanding and a predictable theory to unravel the effects of geometry on the mechanical behavior of the metamaterials with arbitrary sizes of the alternatingly large and small holes. In experiments, to better control the deformation, we design and fabricate pneumatically actuated metamaterials, as illustrated in Fig. 1a, where an elastomeric slab patterned with a square array of holes is sealed by two thin elastomeric membranes. Narrow and shallow grooves are used to connect all the holes so that air can flow freely throughout the whole structures. The holes are further linked to an external source of vacuum via a tube (Fig. 1b). At a critical pressure Δp_{cr}, with Δp defined as the difference between the external and internal pressures, $\Delta p = p_{ext} - p_{int}$, the metamaterial undergoes a pattern transformation with the circular holes altering their shapes into ellipses orthogonal to their neighbors, yielding a large transformation strain (Fig. 1c–d). The pattern transformation, and therefore the corresponding transformation strain, is widely tunable: the pattern of uniform-sized holes leads to an equibiaxial contraction (Fig. 1c), whereas the pattern of alternatingly large and small holes leads to a non-equibiaxial contraction (Fig. 1d). We will numerically and experimentally characterize the pattern transformation as a function of the pressure Δp, and explore the effect of a wide range of geometric parameters. We will further provide an analytical model to explain how the geometry affects the pattern transformation of the metamaterials.

The paper is organized as follows. In Section 2, we perform finite element simulations to investigate the tunable pattern transformation of the metamaterial by surveying the design space of the geometric parameters. In Section 3, we develop an analytical model to understand the effect of geometry on the critical condition for the pattern transformation, the transformation strain and the transformation type. Finally, we conduct experiments in Section 4 to demonstrate the tunable pattern transformation of the metamaterials and verify the results of the finite element simulations and the analytical model. Section 5 will conclude the paper.

2. Finite element analysis

In this section, we systematically investigate the role of geometry in determining the critical pressure, transformation strain and the transformation type using finite element simulations in Abaqus (version 6.14).

We perform simulations in a unit cell of an elastomeric slab (Fig. 2a), which is selected as the smallest geometric unit that can build up the whole system by mirroring and patterning itself [34]. Symmetric boundary conditions are applied to the unit cell, i.e., on the bottom and left boundaries displacement is only allowed in the horizontal and vertical directions, respectively, and the top and right boundaries are constrained to maintain horizontal and vertical, respectively (Fig. 2b). Although these boundary conditions are not periodic, a 2 × 2 cell, by mirroring the current unit cell horizontally and vertically, is equivalent to the smallest repeated structure for the periodic boundary condition. Besides, the unit cell is subjected to a pressure Δp on all the surfaces of the holes. The geometric parameters of the unit cell include the two radii R and r, and the lattice size L, the subtraction of which defines the thickness of the slenderest wall, $t = L - R - r$. We normalize the geometric parameters by the lattice size L and obtain two independent dimensionless parameters: R/L and r/L. We model the elastomer as an incompressible neo-Hookean continuum solid, whose strain energy density function W is given as [35,36]

$$W = \frac{\mu}{2}\left(tr\left(\mathbf{FF}^{\mathrm{T}}\right) - 3\right) \tag{1}$$

where μ is the shear modulus, and $\mathbf{F}$ is the deformation gradient tensor with the constraint det($\mathbf{F}$) = 1. We use eight-node, quadratic, hybrid, plane strain elements (ABAQUS element type CPE8H), and perform mesh refinement study to ascertain the accuracy of simulations. To break the symmetry, small imperfections with amplitude $\sim t/1000$ are introduced to the initial coordinates of the nodes. We simulate the deformation of the unit cell under the pressure based on the Riks method, and calculate the strains ε_x and ε_y, defined as the changes in length of the lattice with respect to the original length in the x and y directions (Fig. 2c).

Fig. 3 shows the simulation results of the strain–pressure relations (Fig. 3a, b) and the shape evolution of four unit cells of