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Impact of node geometry on the effective stiffness of non-slender three-dimensional truss lattice architectures

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ABSTRACT

Three-dimensional (3D), lattice-based micro- and nano-architected materials can possess desirable mechanical properties that are unattainable by homogeneous materials. Manufacturing these so-called structural metamaterials at the nano- and microscales typically results in non-slender architectures (e.g., struts with a high radius-to-length ratio r/l), for which simple analytical and computational tools are inapplicable since they fail to capture the effects of nodes at strut junctions. We report a detailed analysis that quantifies the effect of nodes on the effective Young's modulus (E^*) of lattice architectures with different unit cell geometries through (*i*) simple analytical constructions, (*ii*) reduced-order computational models, and (*iii*) experiments at the milli- and micrometer scales. The computational models of variable-node lattice architectures match the effective stiffness obtained from experiments and incur computational cost that are three orders-of-magnitude lower than alternative, conventional methods. We highlight a difference in the contribution of nodes to rigid versus non-rigid architectures and propose an extension to the classical stiffness scaling laws of the form $E^* \propto C_1(r/l)^\alpha + C_2(r/l)^\beta$, which holds for slender and non-slender beam-based architectures, where constants C_1 and C_2 change with lattice geometry. We find the optimal scaling exponents for rigid architectures to be $\alpha = 2$ and $\beta = 4$, and $\alpha = 4$ and $\beta = 6$ for non-rigid architectures. These analytical, computational, and experimental results quantify the specific contribution of nodes to the effective stiffness of beam-based architectures and highlight the necessity of incorporating their effects into calculations of the structural stiffness. This work provides new, efficient tools that accurately capture the mechanics and physics of strut junctions in 3D beam-based architected materials.

1. Introduction

Architecting structural metamaterials at the micro- and nanoscales has enabled the attainment of novel mechanical property combinations that are inaccessible to homogeneous materials. For example, decoupling high stiffness and high strength from density [1–7], eliciting a phononic bandgap response [8], and twist-to-compression coupling [9] have been made possible due to the use of architecture. Besides unveiling interesting material behavior at the nanoscale, these types of structural metamaterials exploit a careful structural design to achieve their unique properties [5,10–12]. For instance, optimization of structural elements has enabled the fabrication of multistable structures for strain energy storage [13–15] as well as large-deformation energy absorption [16–18]. Benefits of architecture have also been exploited at larger scales than the ones described above, such as in centimeter-scale metallic and composite sandwich plates with truss architectures instead of metallic foams or corrugations, where the architected core elicits superior performance [19,20]. Due to their high mechanical tunability, architected structural materials are sometimes subjected to unusual loading conditions such as in cell scaffolds [21,22], truss-based micro-battery lithiation [23], and impact environments [16,24,25], all of which demand developing a full understanding of the mechanical properties at each relevant length scale. Several works have contributed to this understanding by exploring the mechanical parameter spaces of some architected materials [2,4–6,26], which successfully linked their high stiffness, strength, and resilience to the underlying structural hierarchy and shell-based geometries. However, substantial unattained properties still remain within those parameter maps and few overarching guidelines have been developed to accurately characterize architected materials across scales and structural parameters. This understanding of the micro-architecture across wider parameter spaces is crucial to enabling materials by design.

Recent developments in the field of architected materials include the fabrication of millimeter- to centimeter-scale volumes of

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material while maintaining nano- and microscale feature resolution [27]. Several earlier works exposed unique properties enabled by the material–structure interaction at these scales [2–5,11] but were limited to relatively small tessellations of periodic architectures to result in small sample dimensions. The new opportunity of fabricating much larger tessellations enables designing such metamaterials for structural applications, which demand effective analytical and computational tools to predict a given structural member's mechanical properties and response to prescribed loading.

An extensive experimental and theoretical body of literature has explored the effective, macroscale properties of lattice architectures, which has laid the basis for analytical predictive tools [28–33]. For example, using beam theory, the classical stiffness and strength scaling laws by Gibson and Ashby [34] have related a cellular solid's effective Young's modulus (E^*) and strength (σ_y^*) to its relative density ($\bar{\rho}$):

$$\frac{E^*}{E_s} = C\bar{\rho}^{\xi}, \tag{1}$$

$$\frac{\sigma_y^*}{\sigma_{ys}} = D\bar{\rho}^{\eta}, \tag{2}$$

where E_s is the constituent material's Young's modulus, σ_{ys} is the constituent material's yield or failure strength, C and D are proportionality constants, and ξ and η are the scaling exponents. The proportionality constants C and D as well as the scaling exponents ξ and η vary with the type of architecture.

Lattice architectures can be broadly characterized based on their rigidity, which directly determines the scaling exponents. A necessary but not sufficient condition for rigidity is provided by Maxwell's condition for pin-jointed structures [35], which is based on the connectivity of a given structure, i.e., the number of bars and joints that form the structure. It states that a three-dimensional structure with b bars and j joints is rigid if $b - 3j + 6 = 0$. A more complete analysis requires assembling the equilibrium matrix for a given unit cell and analyzing its fundamental subspaces to determine the number of states of self-stress s and zero-energy mechanisms m, as shown by Pellegrino and Calladine [36]. The existence or lack of a zero-energy mechanism classifies a structure as rigid ($m = 0$), non-rigid ($m > 0$) [37], or as periodically rigid ($m = 0$ when periodic boundary conditions are applied to the unit cell). Rigid architectures possess stretching-dominated behavior while non-rigid ones predominantly undergo bending-dominated deformation, but may also exhibit stretching-dominated behavior in some loading directions due to anisotropy (e.g., simple-cubic lattice). Assuming rigid structures are probed in a stretching-dominated direction and non-rigid ones in a bending-dominated direction, beam theory predicts $\xi = 1$ and $\xi = 2$ in Eq. (1), respectively.

Some works have experimentally demonstrated these classical stiffness scalings [2,11], while others have reported experimental and computational stiffness scalings that differ from these beam-theory predictions [1,3,11,16,32]. The non-slender beams that comprise some of the cellular architectures probed experimentally may explain the observed deviations from the classical models, since beam theory does not offer a suitable approximation for those structures. In fact, the computational studies of Meza et al. [38] confirmed, for specific lattice architectures, that the classical predictions apply if the struts are slender, i.e., $(r/l) \lesssim 0.05$, and that a transition to a different scaling is observed as the struts become non-slender. Without quantification, that work identified the nodes as the main cause for the deviation from classical scaling. Lattice architectures in many small-scale experimental studies have beam dimensions that render the beam-theory approximation inapplicable. For instance, slender solid-beam lattices, (i.e., ones with $(r/l) \lesssim 0.05$) are challenging to fabricate at the nano- to microscales due to fabrication constraints [38]. This implies that the deviation from the classical scaling and the absence of a global scaling law pose a challenge for designing these types of architected materials. This geometrical regime, at which the existing theoretical tools cannot capture the mechanics of lattice architectures, drives the need for developing advanced theories and numerical tools.

Effective and efficient numerical tools have become increasingly important as the limits of analytical prediction tools have been approached. For example, discrete beam element [39,40] and beam homogenization models [41–46] are effective only for slender-strut architectures and are not capable of capturing the physics of complex, non-slender architectures such as the thin-walled, hollow-tube ceramic lattices of Meza et al. [11] or the metallic lattices of Schaedler et al. [1]. Alternative models, such as full-scale 3D finite element (FE) models, accurately capture the physics of lattice architectures but pose additional challenges due to high computational costs. Applying periodic boundary conditions to a unit cell has been widely utilized as an effective approach to reduce the computational cost of these models [26,38], but it is inadequate when predicting the behavior of finite tessellations due to boundary effects or non-infinite boundary conditions [32]. Alternatively, the full tessellation can be modeled, which retains the full detail of the architecture and the boundary conditions but requires a large amount of computational resources [16,17].

Here, we present a comprehensive study that investigates and quantifies the effect of node geometry on the stiffness scaling of non-slender lattice architectures using (i) simple theoretical approximations for nodes, (ii) reduced-order computational models where full detail at the nodes is retained, and (iii) experiments at the milli- and micrometer scales with varying node geometries. We discover that the nodes in non-rigid architectures have greater contribution to stiffness than those in rigid ones, determined by efficient reduced-order models and experiments at multiple length scales, and predicted by simple theoretical constructions. We demonstrate that the reduced-order models provide an efficient alternative to obtain stiffness properties of full lattice tessellations, to within a few percent error compared to full-resolution models with up to three orders-of-magnitude reduction in computational cost. We propose an extension to the classical stiffness scaling laws that enables them to effectively describe the scaling of both slender and non-slender lattice architectures and account for the effects of nodes and their geometry.

2. Theory

We begin with simple, representative 2D beam-element networks of rigid and non-rigid geometries to provide intuition on their stiffness scaling as a function of beam slenderness. Using linear two-node Euler–Bernoulli beam elements with circular cross-sections (radius r, modulus E, and length l), we define the beam slenderness as the radius-to-length (r/l) ratio. To account for the effect of the nodes, i.e., the beam junctions, we utilize variable-stiffness rotational springs at junctions in addition to beam elements. With at most two beam elements connected at a given junction in the model, each junction has two translational and two rotational degrees of freedom, the latter of which are coupled by a rotational spring with stiffness k_θ (Fig. 1(a)). The total strain energy density of a structure with n_b beams and n_θ rotational springs assembled in this manner takes the form

$$W_{\text{tot}} = \sum_{i=1}^{n_b} W_b^i + \sum_{j=1}^{n_\theta} W_\theta^j, \tag{3}$$

where W_b^i and W_θ^j are the energy densities of the ith linear Euler–Bernoulli beam and the jth rotational spring, respectively. The