Extreme Mechanics Letters 18 (2018) 70–78

Contents lists available at ScienceDirect

Extreme Mechanics Letters

journal homepage: www.elsevier.com/locate/eml

Depletion-induced extreme plasticity across multiple horizontal wellbores in reservoirs with soft formations

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ARTICLE INFO

Article history:
Received 5 July 2017
Received in revised form 9 November 2017
Accepted 10 November 2017
Available online 21 November 2017

ABSTRACT

Following depletion of oil and gas reservoirs with soft rocks, serious wellbore damage and a large amount of sand production result from the development of extremely large plastic deformation around wellbores. Evaluation on the development of plastic strain is crucial to wellbore instability analysis and sand production forecasting. Cap plasticity models have been developed to capture both shear and compaction failures for geomaterials. However, it has been a great challenge in accurately identifying extremely large plastic deformation around wellbores in reservoirs with soft formations using cap plasticity model coupled with fluid flow. To tackle this challenge, we have developed a fully implicit and fully coupled geomechanics and fluid flow finite element code where the Pelessone smooth cap plasticity model is consistently formulated for finite plastic deformation. In this work, with this robust and accurate computer code, we successfully investigate a detailed evolution of plastic strain up to its extreme condition around and across multiple horizontal wellbores following reservoir pore pressure drawdown.

1. Introduction

A large number of oil and gas reservoirs embodies geologically young and poorly consolidated geomaterials with high porosity. For these reservoirs with soft formations, serious issues in terms of wellbore instability, sand production, and ground surface subsidence resulting from reservoir depletion have been major concerns in petroleum industrials and environmental agencies. For example, more than 1000 production wells in the Belridge oil and gas field reservoirs with high porosity diatomite rocks in California [1] and more than 90 wells in chalk reservoirs in the North Sea experienced severe borehole damage and casing failure resulting from depletion-induced compaction of soft formations. On the other hand, ground surfaces over these reservoirs subsided more than 30 ft. These subsidence issues have been negatively impacting environments and offshore platform operations. Understanding the mechanism of formation failures, developing mathematical and physical models, and predicting reservoir physical events are crucial for wellbore instability analysis and sand production management for reservoirs with soft formations.

Traditional shear-dominated failure models such as the Drucker–Prager plasticity model [2] are not able to predict the failure due to compaction of geomaterials. The mechanism of compaction failure is based on the collapse of rock pores from the micromechanical point of view. To capture such a compaction failure mode, a cap plasticity model [3] was developed by introducing a cap to enclose an opened cone-like yield surface of the Drucker–Prager plasticity model. This cap plasticity yield surface has a sharp corner point that may bring a difficulty for numerical modeling with regard to convergence issues. To avoid these issues, a few mathematically more robust cap models with smooth transition between shear surface and compaction surface were proposed by [4–9] for geomaterials, concretes, and metal powders. To capture both dilatant and compaction failure modes, various smooth cap models were calibrated through laboratory experiments for various rock formations with high porosity [10,11].

Due to complexity of cap plasticity models, numerical methods must be adopted to solve cap plasticity problems numerically for practical applications. For pure solid materials, Sander et al. [3], Hofstetter et al. [12], and Simo et al. [13] presented implicit finite element formulations for solving non-smooth cap models. A few implicit and consistently formulated material integrators for smooth cap models were derived in [14–18] for geomaterials undergoing small plastic deformation. Hartmann [9] derived a consistent finite element formulation for metal powders undergoing finite plasticity. For coupled geomechanics with fluid flow, Borja et al. [19] extended a consistent finite element formulation for cam-clay plasticity model for small deformation to finite strain cases. The essence of these numerical formulations is to consistently construct material integrators which enforce the exact condition of stresses on yielding surfaces, update stresses, and provide algorithmic tangent modulus for global Newton–Raphson

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https://doi.org/10.1016/j.eml.2017.11.004
2352-4316/

iterations. With consistently constructed material integrators, high fidelity simulations with quadratic convergent rates in nonlinear iterations are highly expected for these formulations in solving practical plasticity problems.

The objective of this work is to numerically study the evolutional profile of extremely large plastic strains developed across multiple horizontal wellbores following the depletion of reservoirs with soft formations. Identification of such an evolutional profile of plastic strain is very important to understand how formation fails across multiple wellbores. The distinctive feature of this work is to perform complex multiple wellbore analysis in a fully coupled and fully implicit computational framework consistently formulated for finite cap plasticity. The ability of cap plasticity models at predicting large subsidence of reservoirs in the North Sea up to 30 ft was demonstrated in [20] where only a pure solid equation was solved without considering wells and fluid flow. An elegant and rigorous consistent material integrator was formulated for the Pelessone smooth cap model by [16] for pure solid field with small deformation but only simple problems were tested and reported. While plastic strains around a single wellbore was studied using a smooth cap plasticity model formulated in finite hyperelasto-plasticity framework by [21], only pure solid field equation was solved and maximum volume plastic strain was only in the level of three percentage. A strip footing problem with punching loading was solved in a fully coupled geomechanics and fluid flow framework using cap plasticity model by [22]. In [22], to achieve convergent solutions a semi-implicit formulation was adopted for material integrators for cap models to avoid numerical instability. However, stresses from semi-implicit formulations are not in exact stress spaces and solutions may have large errors for long term predictions. For reservoir problems, geomechanical solutions are sensitive to fluid flow around boreholes. Furthermore, unlike a single wellbore drilled in a large pay zone, the evolution of pore pressures across multiple boreholes is very complex, which greatly affects the development of plastic strain in solid skeletons near boreholes. In this paper, we developed a fully coupled geomechanics and fluid flow computational framework where we consistently constructed a material integrator based on the Pelessone smooth cap model and consistently linearized coupled solid and fluid flow equations for finite plastic deformation. With this fully coupled and fully implicit code, we are able to accurately identify the evolution of plastic strain across multiple horizontal wellbores following the drawdown of reservoir pore pressure.

We outline the remaining sections of this paper as follows. In Section 2, we present two coupled field equations based on Biot's consolidation theory. The Pelessone smooth cap plasticity model is introduced in Section 3. We briefly summarize our finite element formulation and nonlinear procedures for solving coupled geomechanics and fluid flow problems in Section 4. In Section 5, we define a reservoir model with multiple horizontal wellbores. Specifically, careful enforcements on initial conditions and bore hole pressure drawdown for models with soft formations are further addressed. Finally, we present and discuss the predicted evolution profile of large plastic strains across multiple wellbores following the depletion of reservoirs. We end with conclusions.

2. Coupled geomechanics and fluid flow problems

The governing equations for coupled geomechanics and single phase fluid flow problems are defined as follows [23–25]:

$$\begin{cases} \nabla\cdot\sigma(x,t)+f(x,t)=0 & (x,t)\in\ \Omega\times(0,\infty) \\ \alpha\dot{\epsilon}_v(x,t)+\dfrac{1}{M(x,t)}\dot{p}(x,t)+\nabla\cdot q(x,t)=s_q(x,t) & (x,t)\in\ \Omega\times(0,\infty) \end{cases} \tag{1}$$

where σ, p, q, and s_q are the Cauchy stress, pore pressure, flow velocity, and fluid flow source. In Eq. (1), parameters α and M are Biot's constant and modulus. For solid skeleton undergoing deformation, porosity is affected by the strain of solid skeleton. Therefore, α and M can change following solid deformation. f in the first equation in Eq. (1) is the body force of porous media defined as $f(x,t)=[(1-\phi)\rho_s+\phi\rho_f]g$ where ϕ, ρ_s, ρ_f, and g are the porosity of the porous medium, the density of solid skeleton, the density of fluid, and the acceleration by gravity. Boundary conditions for Eq. (1) are prescribed as follows:

$$\begin{cases} u(x,t)=\bar{u}(x,t) & x\in\Gamma^{u}\times(0,\infty); \\ \sigma(x,t)n(x,t)=t_s(x,t) & x\in\Gamma^{t/w}\times(0,\infty); \\ \sigma(x,t)n(x,t)=-\bar{p}(x,t)n(x,t) & x\in\Gamma^{w}\times(0,\infty); \\ p(x,t)=\bar{p}(x,t) & x\in\Gamma^{p}\times(0,\infty); \\ q(x,t)\cdot n(x,t)=t_q(x,t) & x\in\Gamma^{q}\times(0,\infty) \end{cases} \tag{2}$$

where superscripts u, t, p, q, and w on boundary Γ indicate the boundary portions corresponding to displacement, traction, pore pressure, fluid flux, and traction on borehole wall surfaces. Specifically, for reservoir problems, vertical overburden is applied in models through the second equation in Eq. (2) and controlled borehole depletion pressure is enforced through the third and fourth equations in Eq. (2), i.e. the pressure loading on borehole wall surfaces for solid equation and the primary pore pressure boundary condition for fluid flow equation. Initial conditions in terms of total Cauchy stress and pore pressure are given below:

$$\begin{cases} \sigma(x,t)_{t=0}=\sigma_0(x) & x\in\Omega \\ p(x,t)_{t=0}=p_0(x) & x\in\Omega. \end{cases} \tag{3}$$

Several auxiliary equations are needed to complete Eq. (1). First, the relation of the effective stress (σ'') and total stress is given by $\sigma''=\sigma+\alpha pI$. Second, kinematic condition for infinitesimal strain in solids is described as $\epsilon=\frac{1}{2}(\nabla u+\nabla^T u)$ where ϵ is the total strain and notation ∇ is the gradient operator. Third, the constitutive law for solid skeleton is governed by Hooke's law as $\sigma=\mathcal{C}^e\epsilon^e=\mathcal{C}^e(\epsilon-\epsilon^p)$ where $\mathcal{C}^e$ is the elasticity tensor, ϵ^e the elastic strain tensor, and ϵ^p the plastic strain tensor. Fourth, the rate change of plastic strain is determined by plastic potentials as $\dot{\epsilon}^p=\lambda\frac{\partial\mathcal{F}}{\partial\sigma}$ where $\mathcal{F}$ is the plastic flow potential, and λ is related to material yield condition: $\lambda>0$ if $Y=0$ and $\lambda=0$ if $Y<0$. Finally, Darcy's law for fluid flow in porous media is defined as $q=-\frac{\mathcal{K}}{\mu}(\nabla p-\phi\rho_f g)$ where q is the velocity of fluid flow and $\mathcal{K}$ is the permeability tensor of the porous medium and μ is the viscosity of the fluid phase.

3. Cap plasticity model

Three invariants of the Cauchy stress and the deviatoric stress are defined as $I_1=trace(\sigma)$, $s=\sigma-\frac{I_1}{3}I$, $J_2=\frac{1}{2}s:s$, and $J_3=det(s)$ where I_1, J_2, and J_3 are the first invariant of the Cauchy stress, second and third invariants of the deviatoric Cauchy stress, and s the deviatoric stress. In this study, we implemented the Pelessone smooth cap model [4] with isotropic hardening models for both shear and compaction yield surfaces. The yield function of Pelessone's smooth cap model is defined as follows:

$$Y(I_1,J_2,J_3,\sigma_c,X)=\Gamma^2(\beta)J_2-Y_c(I_1,X,K(X,\sigma_c))Y_s^2(I_1,\sigma_c) \tag{4}$$

where Y_s is a function to define the yield surface of the shear portion as follows:

$$Y_s(I_1,\sigma_c)=\sigma_c-A_s^Y(e^{B_s^Y I_1}-1)-\alpha_s^Y I_1 \tag{5}$$

where σ_c is the cohesive strength, α_s^Y is related to frictional angle of geomaterials, and A_s^Y and B_s^Y are used to address yield surfaces calibrated from experimental data which may deviate from linear behaviors, and the superscript Y indicates a variable or a constant related to yield function. Y_c in Eq. (4) is an elliptic function associated with the cap portion of the yield surface and it is defined as