International Journal of Adhesion and Adhesives 82 (2018) 14–20

Contents lists available at ScienceDirect

International Journal of Adhesion and Adhesives

journal homepage: www.elsevier.com/locate/ijadhadh

Interlaminar shear stress function for adhesively bonded multi-layer metal laminates

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ARTICLE INFO

Keywords:
Adhesive bonding
Laminates
Interlaminar shear stress
Load transfer efficiency

ABSTRACT

A new analytical model is proposed for the estimation of interlaminar shear stress in adhesively bonded metal laminates consisting of an arbitrary number of layers. The interface shear stress in the laminates is related to the difference in average axial strain and elongation between adjoining layers through a newly proposed interlaminar shear stress function (ILSSF). The parameters of the ILSSF are determined from finite element simulations using a data fitting procedure. The accuracy of the model is investigated by comparing experimental measurements of average elongation in three-layer aluminum laminates to values obtained using the model. Good agreement with the experimental results is achieved for several types of adhesives and for different ratios of adhesive-to-layer thicknesses. The influence of Young's modulus of the adhesive on the efficiency of load transfer in three-layer laminates is investigated.

1. Introduction

Adhesively bonded laminates are used for engineering applications in fields as diverse as construction, aircraft and automotive manufacturing, energy harvesting, etc. This widespread use is primarily motivated by the good mechanical properties achievable by these laminates [1]. A relatively recent application for laminates is in the field of energy harvesting, wherein adhesively bonded laminates usually comprise layers of piezoelectric materials capable of generating an electric charge when deformed. In particular, laminates in the form of morph and bimorph beams were shown to achieve promising results, and are therefore candidates for future energy harvesting applications [2]. The integrity and performance of a laminate is significantly influenced by the efficiency of load transfer at the interface between adjoining layers. In particular, the interlaminar shear stress plays an important role in promoting the efficiency of load transfer, governing failure modes and influencing the overall strength of the laminate [3–5]. In energy harvesting devices utilizing piezoelectric bimorphs, the efficiency of load transfer between metallic and piezoelectric layers strongly influences the strain within individual layers and, therefore, the electric charge generated by means of the piezoelectric effect. The middle layer plays an important role in a bimorph since it is typically subjected to thermal and mechanical loads, which are then transferred to the piezoelectric layers to generate electricity [6]. The classical laminate theory assumes no interlaminar shear stress, resulting in a load transfer efficiency of 100%. This assumption may be inaccurate depending on factors such as the length, thickness, and other mechanical properties of the adhesive and the layers [5]. More accurate models for interlaminar shear stress have been the focus of current researches [7–12]. In particular, Touratier [7] proposed a standard layer theory based on a kinematical approach in which the interlaminar shear was represented by a sinusoidal function. The relevant boundary value problems were then derived using the principle of virtual power and solved by means of numerical methods. The author simulated bending, free undamped vibration, buckling, wave propagation, torsion, etc. Based on the continuity theory for laminates, Lu et al. [8] presented a method to account for interlaminar shear stress continuity and transverse shear deformation. The theory was verified through comparisons with Pagano's elasticity analysis. It was found that the proposed theory allowed excellent estimation of stress and displacement in the laminate. Moreover, the interlaminar shear stress function (ILSS) could be determined directly from constitutive relations instead of being recovered from equilibrium equations. Kassapoglou [9] introduced a method to determine the complete state of stress at the free edges of composite laminates subjected to combined loads (uniaxial tension or compression, bending, and out-of-plane shear) based on the principle of minimum complementary energy. The interlaminar stress was determined in closed form and found to be in good agreement with published results. Ko and Lin [10] investigated interlaminar stresses around a hole in symmetric cross-ply laminates under bending/torsion.

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https://doi.org/10.1016/j.ijadhadh.2017.12.011
Accepted 13 December 2017
Available online 19 December 2017
0143-7496/

The influence of interlaminar shear stress on bending deformation was also investigated [11,12]. The above publications are mainly based on theoretical analysis and computer simulations, which require experimental validation.

In this work, an interlaminar shear stress function is proposed for the determination of shear stress in the interface between two adjoining layers in *n*-layered laminates based on the shear-lag model (SLM). The parameters of the stress function are determined by means of a data fitting procedure using finite element analysis results. Experimental validation is carried out for the case of three-layer aluminum laminates bonded using different types of adhesives and subjected to axial loading. The method proposed herein can be extended to other loading cases, including bending and torsion of laminates with cylindrical and rectangular layered geometries [13].

2. Modeling methods

2.1. Shear-lag model

The SLM is used to describe the behavior of laminates because of its ability to predict the mechanical properties of interfaces and the distribution of stress within. The model has been validated by means of numerical simulations and experiments [14,15].

In this work, SLM and finite element analysis are utilized to describe the interfacial behavior of *n*-layer laminates like the one illustrated in Fig. 1. The layers of width *a*, thickness *t* and length *2L* are bonded using four different types of adhesives, which act as media for load transfer between adjacent layers undergoing relative deformation. As shown in Fig. 1, the adhesive has a width *a*, thickness *s,* and length *2L*. The bonding between the layers and the adhesive is assumed to be perfect in the simulations. Moreover, the tensile strength of the adhesive is assumed to be much lower than that of the metal layers and the load is considered small enough so that joint failure does not take place. Without loss of generality, the external force is assumed to be applied only on a single layer. Axial stress is measured in the axial direction by taking the average stress over the thickness, defined as follows [14,16,17]:

$$\sigma_{am} = \frac{1}{t}\int_0^t \sigma_{k.x}(x, y).\ dy, \tag{1}$$

where $\sigma_{k.x}(x, y)$ is the stress in layer k at point (x, y). An infinitesimal element of length dx is extracted to examine the force equilibrium as shown in Fig. 1, leading to an approximate measure of the average axial stress in the loading direction given by the relations

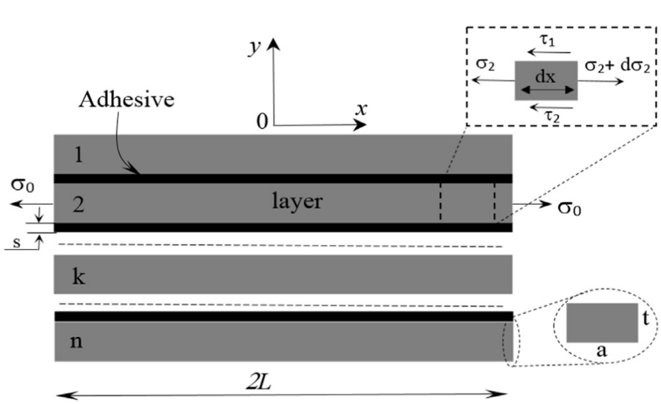

Fig. 1. Shear-lag model for a *n*-layer laminate.

$$\frac{d\sigma_{a1}}{dx} = -\frac{\tau_1}{t},$$
$$\frac{d\sigma_{am}}{dx} = \frac{\tau_k - \tau_{k-1}}{t} = \frac{\tau_{m-1} - \tau_m}{t},\quad with\ m = 1\ \ to\ n\ and\ k = m - 1,$$
$$\frac{d\sigma_{an}}{dx} = \frac{\tau_{n-1}}{t}, \tag{2}$$

where σ_{am} is the average axial stress in layer m and τ_k is the interlaminar shear stress at interface k, respectively. In this work, the interlaminar shear stress is assumed to be described in terms of adhesive properties and the difference in average axial strain between adjacent layers. More specifically, for a particular value of r_a, the following expression is proposed for the interlaminar shear stress:

$$\tau_k = \tau_{m-1} = f(E_a).\ (\varepsilon_{am-1} - \varepsilon_{am}), \tag{3}$$

where ε_{am} is the average axial strain in layer m and E_a is Young's modulus of the adhesive used in FEM, respectively, which will be further discussed in the following section. From observation of the interlayer shear stress in our previous work [5], $f(E_a)$ is proposed to be an exponential function in the form

$$f(E_a) = A.\ e^{B.E_a} + C.\ e^{D.E_a}, \tag{4}$$

where *A, B, C,* and *D* are coefficients. *A* and *C* have the same unit as E_a and *B* and *D* have the inverse unit of E_a. The form reported in Eq. (4) for the function $f(E_a)$ was found to closely fit the observed variation in interlaminar shear stress, which presents an exponential trend, while at the same time being relatively simple to characterize. Further simplification to a single term exponential such as $f(E_a) = A.e^{B.E_a}$ was found to result in excessive error. Substituting Eqs. (4) and (3) into Eq. (2) and interchanging the stress and strain, the following system of differential equations is obtained, which describes the distribution of strain in the laminate:

$$\frac{d\varepsilon_{a1}}{dx} = \frac{(A.\ e^{B.E_a} + C.\ e^{D.E_a})(\varepsilon_{a2} - \varepsilon_{a1})}{E_{l1}t},$$
$$\frac{d\varepsilon_{am}}{dx} = \frac{(A.\ e^{B.E_a} + C.\ e^{D.E_a})(\varepsilon_{am+1} + \varepsilon_{am-1} - 2\varepsilon_{am})}{E_{lm}t},$$
$$\frac{d\varepsilon_{an}}{dx} = \frac{(A.\ e^{B.E_a} + C.\ e^{D.E_a})(\varepsilon_{an-1} - \varepsilon_{an})}{E_{ln}t}, \tag{5}$$

where E_{lm} is Young's modulus of layer *m*. To find the interlaminar shear stress function, the exponential function coefficients *A, B, C,* and *D* need to be determined. This is accomplished by means of finite element analysis of the *n*-layered laminate, which allows the computation of the average axial tip displacements in the layers. The coefficients are then determined by data fitting. It is noted that the interlaminar shear stress is affected by the number of layers and this effect can be identified by solving Eq. (5). Subsequently, the average axial strain in the layers can be determined by substituting Eq. (3) into Eq. (5) and integrating for $x = L$ to $x = 0$ as shown in Fig. 1. The corresponding average axial tip displacement u_{am} is found by integration of the axial strain so that

$$u_{am} = \int_0^L \varepsilon_{am} dx, \tag{6}$$

The initial boundary conditions for the numerical solution of the configuration shown in Fig. 1 are the following:

$$\varepsilon_{a0} = \frac{\sigma_0}{E_{l2}}\ at\ x = L,$$
$$\varepsilon_{am} = 0\ at\ x = L,\quad with\ m = 1, 3, \dots n. \tag{7}$$

2.2. Finite element analysis

In this work, the commercial FEA package ANSYS® Workbench is used to find the average axial tip displacement of the layers of the three-layer laminate shown in Fig. 2, assuming linear elastic behavior. The