Modeling and Model Predictive Control of a thermoacoustically unstable burner

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Abstract: This paper describes the modeling and control of a thermoacoustic unstable burner. The thermoacoustic system is modeled using a 1D-acoustic network coupled with a flame transfer function derived from Computational Fluid Dynamics (CFD). The resulting unstable system provides a realistic test bench for controller evaluation and is controlled using a Model Predictive Controller (MPC) with a sampling time of 0.001s. As QP solver the homotopy approach of qpOASES is used.

Keywords: Thermoacoustic, Instability, Model Predictive Control, Acoustic Network

1. INTRODUCTION

Modern gas turbines have to comply with increasingly stringent emission requirements for NO_x and CO. One of the most effective ways reducing these emissions is the development of Lean Premixed Prevapourised (LPP) combustor systems. The main reason for NO_x production is the combustion temperature. Both, a lean mixture and a premixing of fuel and oxidizer, which increases the mixture homogeneity and therefore avoids temperature peaks in the flame, reduce the combustion temperature. Alongside the aforementioned advantages LPP systems are more susceptible to combustion oscillations than conventional burners, because lean premixed flames are very sensitive to flow fluctuations. In contrast to diffusion controlled flames no stabilizing mixture gradient is present. As a result flow fluctuations can distort the flame easily and consequently the resulting heat release. Fluctuating heat release leads to fluctuating gas expansion. The gas expansion influences acoustic waves traveling through the combustion zone, which in turn influence the flame if reflected. Due to this coupling between acoustics of the combustion chamber and the heat release of the flame a feedback path is established which can give rise to thermoacoustic instabilities (see Fig. 1). A well-known criterion for thermoacoustic instability is the Rayleigh-criterion

$$\int_0^T p'(t) \cdot \dot{q}'(t)\, dt > L \qquad (1)$$

where $p'(t)$ is the acoustic pressure, $\dot{q}'$ the fluctuating rate of heat release and L the acoustic losses. If the integral over one period of oscillation T is positive, the heat release and acoustic pressure interfere constructively meaning that the heat release is in phase with the pressure waves. Thus, the flame feeds energy into the system. If this energy feed is higher than the acoustic losses L the system is unstable and moves into a limit cycle. The limit cycle is a result of saturation effects possibly in the heat release and represents a stable trajectory, where losses and energy feed are in balance. As a consequence high pressure oscillations take place in the combustor which are detrimental for performance, emissions and durability of the combustor components. To avoid these drawbacks two main directions are possible. Passive control techniques include geometry modifications and integration of additional acoustic dampers like Helmholtz resonators. However, the operational range in which stabilization is achieved is limited. To overcome this limitation active control is applied to enlarge the region of stable operation. In addition, existing burners can be equipped as a retrofit. This can be necessary not only for older systems but also for systems, where Computational Fluid Dynamics (CFD) design tools failed to predict instabilities during development.

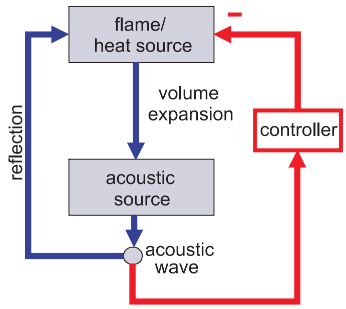

Fig. 1. Thermoacoustic coupling and outer control loop.

2. MODEL

The test rig that is to be controlled is shown in Fig. 2 and consists of a diffusor, a duct, an area discontinuity, a second duct, the flame zone followed by the combustor (duct) and an open end. These components have to be modeled and coupled in order to resemble a complete model, which can be used for control design. The essential parts of the thermoacoustic phenomenon inside the burner are the acoustics and the flame transfer function.

2.1 Acoustics

The acoustic model is a one-dimensional acoustic network consisting of simple geometric components, which are

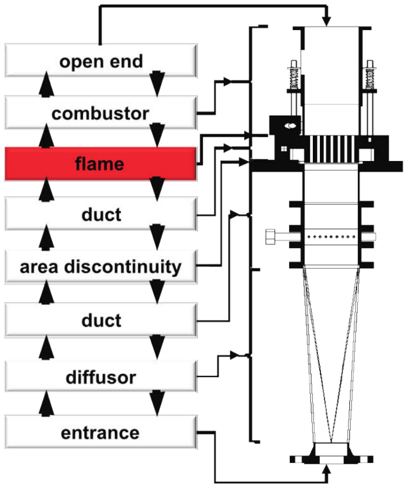

Fig. 2. The different sections of the investigated combustor.

tractable analytically. The underlying assumption, which is common in thermoacoustic community (see e.g. Dowling (1997); Niederberger et al. (2009); Bohn et al. (1993)), is that the mean flow is homogeneous, the length to diameter ratio of the considered elements is sufficiently large such that only axial waves are relevant, the acoustic disturbances are linear and that the flow is isentropic. Then it is sufficient to consider only one-dimensional linear acoustics and the corresponding acoustic state vector (four-pole) is of order $n = 2$. The state can be defined e.g. by acoustic velocity u' and acoustic pressure p'. The total pressure and velocity are defined as the sum of mean and acoustic value as $p(x,t) = \overline{p} + p'(x,t)$ and $u(x,t) = \overline{u} + u'(x,t)$. As shown for example in Poinsot et al. (2005) the solution of the governing conservation equations of mass, momentum and energy lead to the well-known wave equation whose solution under the mentioned assumptions is

$$p(x,t) = \overline{p} + f(t - \frac{x}{\overline{c}+\overline{u}}) + g(t + \frac{x}{\overline{c}+\overline{u}}) \quad (2a)$$

$$u(x,t) = \overline{u} + \frac{1}{\rho\,\overline{c}}\left[f(t - \frac{x}{\overline{c}+\overline{u}}) - g(t + \frac{x}{\overline{c}+\overline{u}})\right] \quad (2b)$$

Here, g and f are the Riemann invariants representing the left and the right traveling wave as can be seen in Fig. 3. Instead of pressure and velocity, we use the Riemann invariants as acoustic states. The advantage is that the resulting transfer functions are always causal if incoming waves are taken as input and outgoing waves as output. As a consequence every element in the one-dimensional network has to connect the Riemann invariants on both sides of the element. For an area discontinuity (see Fig. 3), a mathematical model between f_1, g_2 and f_2, g_1 is needed. According to Munjal (1987) conservation of mass and momentum yields

$$p'_2 = p'_1 \quad (3a)$$

$$u'_2 = \frac{A_1}{A_2}u'_1 \quad (3b)$$

This can be translated into

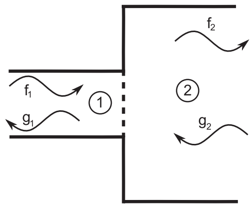

Fig. 3. Shown is a schematic of an area discontinuity.

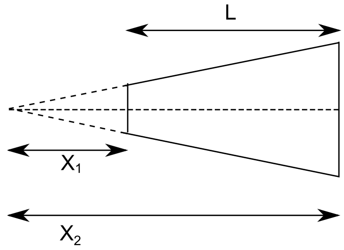

Fig. 4. A conical tube with characteristic dimensions.

$$f_2 = \frac{f_1(A_2 - A_1) + 2A_1 g_2}{A_2 + A_1} \quad (4a)$$

$$g_1 = \frac{2A_2 f_1 + (A_1 - A_2)g_2}{A_2 + A_1} \quad (4b)$$

For the conical tube the following equations mentioned in Bohn et al. (1993) can be used:

$$p'_2 = \frac{1}{x_2}(p'_1(x_1\frac{e^{-sL/c}+e^{sL/c}}{2} + \frac{c}{-s}\frac{e^{-sL/c}-e^{sL/c}}{2}) + u'_1\rho c x_1\frac{e^{-sL/c}-e^{sL/c}}{2}) \quad (5a)$$

$$u'_2 = \frac{1}{x_2}(p'_1(-\frac{L}{x_2\rho s}\frac{e^{-sL/c}+e^{sL/c}}{2} + \frac{c(x_1 s^2/cx_2 - c)}{s^2 x_2\rho}\frac{e^{-sL/c}-e^{sL/c}}{2}) + u'_1(x_1\frac{e^{-sL/c}+e^{sL/c}}{2} + \frac{x_1 c}{sx_2}\frac{e^{-sL/c}-e^{sL/c}}{2})) \quad (5b)$$

where the density ρ and the speed of sound c are assumed constant over the conical tube. x_1, x_2 and L are geometry dependent parameters (see Fig. 4). It is obviously that the corresponding transfer functions are not causal because p'_1, u'_1 as inputs and p'_2, u'_2 as outputs contradict the principle of cause and effect. Relating f_1, g_2 with f_2, g_1 leads to causal transfer functions (see appendix). Finally, to close the system and therefore the path from outgoing to incoming wave boundary conditions for both ends are needed. The end downstream is assumed to be open ended with a reflection coefficient

$$R_d = \frac{-\frac{s^2d^2}{16c^2} - 1 + \frac{0.3sd}{c}}{-\frac{s^2d^2}{16c^2} + 1 + \frac{0.3sd}{c}} \quad (6)$$

given in Munjal (1987). Since the corresponding transfer function is unstable an approximation up to 1500 Hz is used. The upstream boundary condition is assumed to be a choked end (see appendix).

2.2 Flame

Since only low frequencies are of interest the flame zone is short compared to the acoustic wavelength. Thus, the