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Modeling and optimization of the sandwich beam specimen in three-point bending for adhesive bond characterization

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ABSTRACT

Standard tests for adhesive bond characterization suffer for several deficiencies. The simplest specimens to make and test are lap joint geometries (e.g. single, double, symmetric, etc.) that generate complex stress distributions with irregularities and even singularities of the stress state. Those with the stress state closer to pure shear (e.g. napkin ring or Arcan) are difficult to make and require special test fixtures. This paper examines the stress state in the adhesive of a simple beam specimen obtained by bonding two flat plates one upon the other and loading the final sandwich in three-point bending. An elementary theory is used to optimize the specimen for in-situ measurements of either shear strength or shear modulus of the adhesive. The accuracy of the model is validated with finite element analyses, showing good agreement between the analytical and finite element model and also providing suggestions for the best geometry to be adopted for practical implementation of the test.

1. Introduction

Proper design of bonded joints requires reliable information on the mechanical properties of the adhesive in specific application conditions. When adhesives are tested in bulk (neat resin), the measured constitutive properties may not be representative of those in bonded state on three grounds: a) the chemical effect of the adhesive–adherend interphase is lost [1]; b) the adhesive stress triaxiality generated in-situ by the adherends cannot take place [2]; c) the curing conditions may differ because of uncontrolled runaway exothermic reactions in bulk forms [2]. All bond properties are affected [3–5] but the influence on Young's modulus is especially significant [6]. Furthermore, not all adhesive products can be cast as bulk specimens.

Well established tests coupons that include information about the condition of the interface are single, double or other lap specimen configurations [7,8]. However simple to make and test, these geometries have complex stress distributions within the bond and give rise to both normal (peel) and shear stresses that vary from point to point [9,10]. Stress singularities at re-entrant corners and at points of material discontinuity are also an issue [11] that undermine the meaning of these tests for providing genuine strength properties of the adhesive [12–14].

Efforts over the years have been devoted to the development of specimen geometries in which only a single pure shear stress state exists. A notable example in this category is the napkin ring torsion test, either in the standard flat-on-flat configuration [15] or in the cone-on-flat modification [16], which ensures a near-uniform shear stress on the adhesive. A variant of the napkin ring test is the solid butt joint in torsion [2]. This is easier to make than the napkin ring and the Nadai correction gives true stress and strain in the adhesive over the radius. A butt-bonded beam under antisymmetric bending producing a state of simple shear over the bondline was proposed by Wycherley et al. [17] as an adaptation of the famous shear test for metals due to Iosipescu [18]. A uniquely shaped specimen to measure shear stress using an axial tension test machine was introduced by Arcan and coworkers [19]. The only measurements needed in Iosipescu and Arcan tests for shear modulus and shear strength determination are the applied load, the dimensions of the adhesive layer and the relative displacement between the adherends along the bondline. However, elaborate fixtures are needed for proper loading and very precise notches (Iosipescu) and groves (Arcan) must be machined in the specimens to ensure uniformity of stresses and removal of singularities [20]. Also, the application of the load and the measurement of the relative displacement across the adhesive layer are not simple tasks, which require special equipment to be performed correctly. For these reasons, use of these specimens is generally confined to research environments.

Aimed at providing a simple method for routine measurement of adhesive properties, in the late 80s Moussiaux, Cardon and

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Nomenclature

b	Width of beam
C_a	Correction factor for the deflection of the sandwich beam
L	Half-length of the sandwich beam
$(L/h)_{max}$	Minimum between $(L/h)_\delta$ and $(L/h)_\sigma$
$(L/h)_\delta$	Limit aspect ratio to avoid excessive deformation of the sandwich
$(L/h)_\sigma$	Limit aspect ratio to avoid overstress of the adherends
$E,\ G$	Young's modulus and shear modulus of the adherends
$E_a,\ G_a$	Young's modulus and shear modulus of the adhesive
h	Thickness of the adherends
I	Bending moment of inertia of the section of each adherend ($=bh^3/12$)
P	Transverse force acting on beam ends
P_{cr}	Transverse force producing failure of the adhesive
t_a	Thickness of the adhesive layer
$(t_a/h)_{max}$	Max allowed relative thickness of bondline for stress optimization
u_a, u_b, u_s	Axial displacements of adherends and adhesive the interface
$w(x)$	Transverse deflection of beam at position x
x	Distance from beam center along the beam axis
α_L	Relative stiffness of the adhesive layer
$\alpha^*_{L\delta}$	Optimal value of α_L for measurement of the adhesive shear modulus
$\alpha^*_{L\sigma}$	Threshold value of α_L for bond stress optimization
$\gamma_a(x)$	Shear deformation of the adhesive layer
δ	Center deflection of the sandwich beam
δ_0	Center deflection of the homogeneous beam
$\delta_{0\ cr}$	Center deflection of the homogeneous beam under force P_{cr}
$(\delta_{0\ cr}/L)_{lim}$	Limit relative deflection acceptable for the sandwich beam
η, ξ	Dummy coordinates used for integration with respect to x
ν	Poisson's ratio of the adherends
ν_a	Poisson's ratio of the adhesive
σ_{adm}	Admissible working stress of the adherends
σ_b	Normal stress in adherend's cross-section due to bending
σ_{max}	Maximum bending stress in the adherends
$\sigma_{max\ cr}$	Maximum bending stress in the adherends under force P_{cr}
σ_N	Normal stress in adherend's cross-section due to axial force
$\tau(x)$	Shear stress in the adhesive layer
$\tau_{a\ cr}$	Critical stress of the adhesive
τ_0	Maximum shear stress in the homogeneous beam
τ_{max}	Maximum shear stress in the adhesive layer
$\overline{\tau}_{max}$	Limit value of the maximum shear stress in the adhesive layer

Brinson introduced a bonded double cantilever beam specimen [21], which develops a pure shear stress in the adhesive layer. With this easy to make specimen, adhesive properties (shear strength and shear modulus) can be obtained from a standard three-point bend test using the direct readings of force and displacement supplied by the testing machine. With fixed–fixed boundary conditions, the sandwich specimen can also be used in dynamic mechanical thermal analysis (DMTA) systems to obtain viscoelastic shear properties under a range of temperatures and adhesive–adherend interface conditions [22–24].

The basic theory providing the stress distribution in the adhesive and the overall sandwich beam deflection was provided in [21] and further elaborated in [22,23] for different constraint conditions. The theory in this form was assessed in [25] for the beam compliance by means of finite element analyses, showing that it tends to overestimate the beam stiffness especially for low-modulus adhesives and thin adherends. Hunston and coworkers [24] applied the original theory for analyzing the viscoelastic properties of rubbery and glassy adhesives and found encouraging results. A sensitivity analysis of the beam stiffness on the shear modulus of the adhesive was also carried out. An interesting refinement of the theory was proposed by Brinson and coworkers in [26] and used to interpret the results from DMTA testing.

Most papers available on the bonded sandwich beam focus on the determination of the adhesive shear modulus, while the adhesive shear strength is much disregarded. Further, a comprehensive analysis of the merits and the limitations of the test to get information on a wide range of adhesive properties is missing. Using the equations available in [26], this paper aims at optimizing the geometry of the sandwich beam in view of the property to be

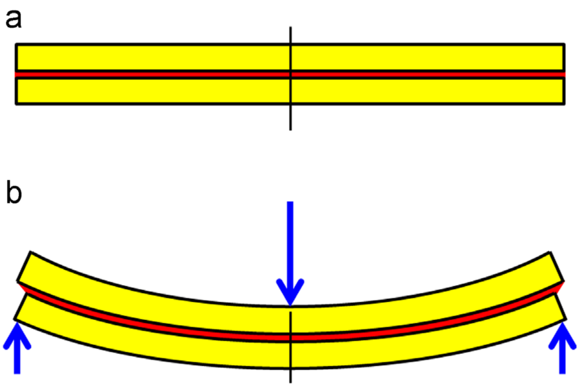

Fig. 1. Schematic of the sandwich beam in the undeformed (a) and deformed (b) configurations.

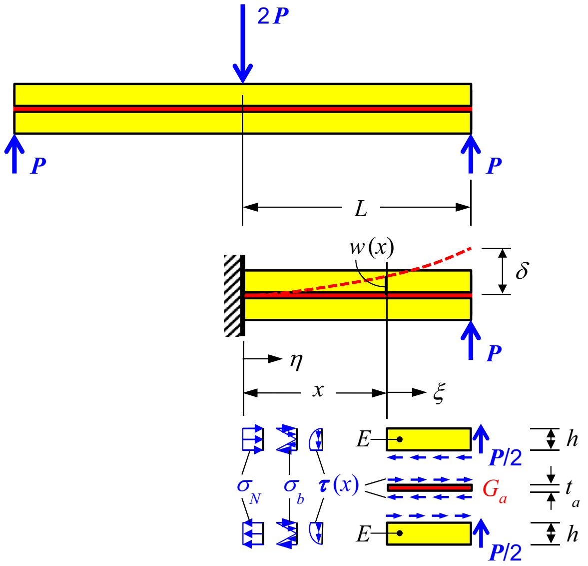

Fig. 2. Bonded sandwich beam and free body diagrams of individual elements with corresponding stress states.

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