International Journal of Adhesion & Adhesives 54 (2014) 112–123

Contents lists available at ScienceDirect

International Journal of Adhesion & Adhesives

journal homepage: www.elsevier.com/locate/ijadhadh

A general solution for the two-dimensional stress analysis of balanced and unbalanced adhesively bonded joints

Zunxu Liu [a], YongAn Huang [a,*], Zhouping Yin [a,*], Stefano Bennati [b], Paolo S. Valvo [b]

[a] State Key Laboratory of Digital Manufacturing Equipment and Technology, Huazhong University of Science and Technology, Wuhan 430074, China
[b] Department of Civil and Industrial Engineering, University of Pisa, Largo Lucio Lazzarino, I-56126 Pisa, Italy

ARTICLE INFO

Article history:
Accepted 19 May 2014
Available online 28 May 2014

Keywords:
Stress analysis
Analytical solution
Adhesively bonded joints
Stiffened plate/joint
Single-strap joint
Single-lap joint

ABSTRACT

This paper presents an efficient analytical solution strategy to determine the adhesive stresses in balanced and unbalanced adhesively bonded joints with mixed force loading and/or displacement boundary conditions. The adhesive stresses are expressed in terms of geometrical dimensions and material properties, combined with integration constants obtained numerically. The model is successfully applied for the analysis of various types of joints, including balanced and unbalanced stiffened plate/joint, single-strap joint, and single-lap joint. In all such cases, the linear equation sets are supplied to determine the integration constants in the final stress expressions. The analytical predictions agree well with the finite element results for adhesive stresses. This proposed model can be extended conveniently to predict the mechanical behavior of similar bonded structures such as composite laminates, electronics packaging, and flexible electronics structures.

1. Introduction

Adhesively bonded joints have found extensive applications for load transfer and connection elements in aerospace, mechanical, and civil engineering structures [1], for structural repairing [2] and for surface-bonded piezoelectric smart structures [3]. Recently, they play increasingly important roles in flexible electronics design [4] and microelectronics packaging [5,6]. Adhesive joints generally consist of three layers: two adherends and an adhesive layer. Due to the discontinuity of the layers at the edges, complicated stress fields and high stress concentrations usually occur in the vicinity of the corners of the adhesive layers. Such high adhesive stresses often result in local yielding of the adhesive and crack propagations in the adhesive or at the adhesive/adherend interface, and may finally lead to the overall failure of the joint. Therefore, efficient and reasonably accurate estimation of the stress level in adhesively bonded joints is crucial for joint design and structural safety evaluation as well as for a better understanding of the underlying mechanisms of failure.

Over the decades, there has been a wide body of the literature on the analysis of adhesively bonded joints typically including, in particular, stiffened plate/joint [5–8], single-strap joint [8–10], and single-lap joint [11–20]. Detailed reviews on the historical development of both analytical models and a finite element method (FEM) for stress analysis of joints can be found in the recent review papers by da Silva et al. [21,22] and He [23]. The overwhelming majority of the aforementioned works focus on the singe-lap joint configuration. The pioneering works by Goland and Reissner [11] furnished the classical solution for the adhesive stresses in joints subjected to mechanical loads. Further improvements have been presented by Hart-Smith [12], Adams and Mallick [16], Tsai et al. [17,18], etc. In an attempt to rationalize the analysis of bonded joints under pure force loading, Bigwood and Crocombe further derived a general elastic analysis [24]. In addition, a large number of experimental studies involving the strength predictions of the single-lap joints were made as well [25–29]. The stiffened and single-strap joint configurations, however, have received little attention. Relevant work on the deformation analysis of the singe-strap joint has been carried out by Shahin and Taheri [9], Li [10], etc. By comparing the reported models, it is found that with releasing the assumptions, the governing equations become increasingly complicated, so that it is challengeable to obtain closed-form expressions for the solution to the differential problem. In such cases, one strategy is to obtain approximate closed-form solutions for relatively simple and/or extremely simplified structures. Another strategy is to solve numerically the differential equations. A similar modelling approach can be adopted to study the analogous problem of the delamination of composite laminates [30]. Accordingly, a delaminated laminate is modelled as an

* Corresponding authors. Tel.: +86 13545354545; fax: +86 27 87543072.
E-mail addresses: yahuang@hust.edu.cn (Y. Huang), yinzhp@mail.hust.edu.cn (Z. Yin).

http://dx.doi.org/10.1016/j.ijadhadh.2014.05.011
0143-7496/

assemblage of sublaminates, connected by a deformable (in most cases, a linearly elastic) interlaminar interface. In such cases, the interface is not necessarily representative of a physical adhesive layer, but is introduced into the model to take into account conventionally the transverse deformability of the laminates, which is neglected in models based on Euler–Bernoulli's simple beam theory. The authors have used this modelling approach to develop an enhanced beam-theory (EBT) model of the asymmetric double cantilever beam (ADCB) test, for which a numerical-analytical solution strategy has been proposed [31]. Recently, an EBT model has been developed also for the mixed-mode bending (MMB) test [32,33]. Furthermore, the model has been extended to general layered structures, showing how the same analytical solution can be used to describe both delaminated laminates and joints [34].

Actually, even though adhesively bonded joints have been intensively investigated over the past decades, more attentions were paid to modelling the overlap region with pure force loading at the adherend ends, namely, consisting of tensile and shear forces and bending moment components. In this paper, we present a general mechanical model and a related solution strategy, which are expected to provide an efficient methodology to model different bonded joints under mixed (force and/or displacement) boundary conditions. It is possible not only to obtain the analytical expressions for the adhesive stresses but also internal forces, displacements, strain and stress components in the adherends. The layout of the paper is as follows. Section 2 describes the mechanical model and provides the theoretical framework of our solution strategy for general adhesively bonded joints. Then the coupled differential problem is solved by conveniently assuming the adhesive stresses as the main unknowns, to obtain a complete analytical solution for the adhesive stresses, internal forces and displacements. Section 3 describes how to apply the force and/or displacement boundary conditions to obtain the integration constants for various balanced and unbalanced adhesively bonded joints, including the stiffened plate/joint, single-strap joint, and single-lap joint. Lastly, in Section 4 the effectiveness and accuracy of the proposed solution strategy are verified by comparison of the predicted adhesive stresses with those computed by using the FEM and other analytical solutions of the literature. Finally, Section 5 draws the conclusions.

2. Formulation of the problem

2.1. Adhesive model in the overlap joints

Generally speaking, overlap joints consist of two slender adherend layers, whose materials may be identical or different from each other, bonded through a thin adhesive layer as sketched in Fig. 1(a). The thicknesses of the upper and lower adherends are denoted by H_1 and H_2, respectively, and the thickness of the adhesive layer in between is h_a, with $h_a \ll H_1, H_2$. Considering a linear elastic body, the materials are assumed isotropic, and the corresponding elastic modulus and Poisson's ratio are E_1, ν_1, and E_2, ν_2 for the adherends, and E_a, ν_a for the adhesive. Local coordinate systems are fixed with their origins at the mid-planes of both adherends. The x-coordinate measures the distance in the axial direction, while local axes z_1 and z_2 denote the distances in the transverse direction from the mid-planes of adherends 1 and 2, respectively. Accordingly, we indicate with u_i and w_i the mid-plane displacements of the substrates along the axial and transverse directions, respectively, and with ϕ_i the rotations of their cross sections, positive if counter-clockwise (here, and in the following, $i=1, 2$ refer to the upper and lower adherends, respectively). In what follows, we consider a unit width in the y-direction

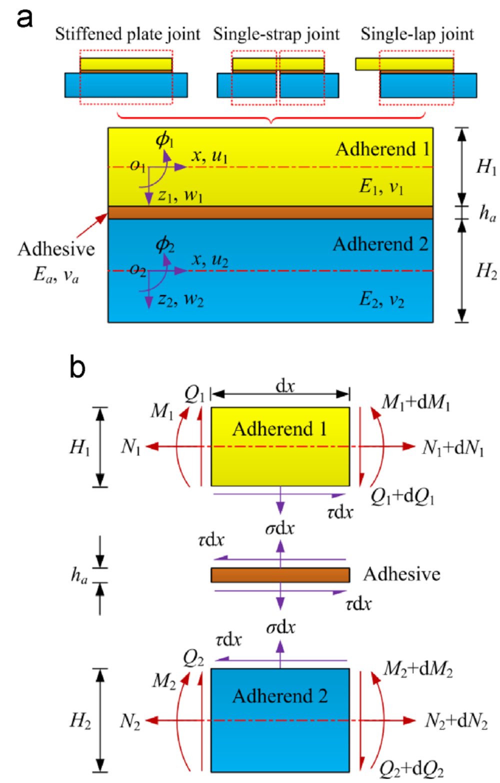

Fig. 1. (a) Typical adhesively bonded joints and corresponding overlap region; and (b) free-body diagrams of adherend and adhesive infinitesimal elements.

(namely, normal to the plane of the figure) and assume plane strain conditions for all the elastic elements involved.

According to Timoshenko's beam theory the axial and transverse displacements at a point inside the adherends are given by $U_i(x,z_i)=u_i(x)+z_i\phi_i(x)$ and $W_i(x,z_i)=w_i(x)$. As mentioned, we neglect any variation of the stresses and strains in the adhesive layer along the z-direction. In particular, the strain components at a point in the adhesive are approximated by their mean values computed from the relative displacements at the top and bottom surfaces of the adherends. Hence:

$$\begin{cases} \varepsilon^a_{zz} = \dfrac{W_2^{\text{top}} - W_1^{\text{bottom}}}{h_a} = \dfrac{W_2|_{z_2=-h_2} - W_1|_{z_1=h_1}}{h_a} = \dfrac{w_2 - w_1}{h_a}, \\ \gamma^a_{xz} = \dfrac{U_2^{\text{top}} - U_1^{\text{bottom}}}{h_a} = \dfrac{U_2|_{z_2=-h_2} - U_1|_{z_1=h_1}}{h_a} = \dfrac{u_2 - u_1 - h_1\phi_1 - h_2\phi_2}{h_a}, \end{cases} \tag{1}$$

where $h_1=H_1/2$ and $h_2=H_2/2$ are the half-thicknesses of the adherends. Under plane strain conditions, if we assume that the adhesive longitudinal normal stress is negligible, only transverse normal (peel) and shear stresses exist in the adhesive. Hooke's law yields the adhesive peel and shear stresses as $\sigma=E_a^*\varepsilon^a_{zz}$ and $\tau=G_a\gamma^a_{xz}$, respectively, where $E_a^*=E_a/(1-\nu_a^2)$ and $G_a=E_a/[2(1+\nu_a)]$ are the elastic modulus (in plane strain) and shear modulus of the adhesive. Based on the aforementioned assumptions, the adhesive layer consists of a uniform, continuous distribution of springs acting in the normal and tangential directions with respect to the interface plane. Accordingly, we define the peel stiffness, k_σ, and shear stiffness, k_τ. A simple, yet effective estimate of the latter constants is given by $k_\sigma=E_a^*/h_a$ and $k_\tau=G_a/h_a$. Therefore:

$$\begin{cases} \sigma = k_\sigma(w_2 - w_1), \\ \tau = k_\tau(u_2 - u_1 - h_1\phi_1 - h_2\phi_2). \end{cases} \tag{2}$$