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A universally applicable multiaxial fatigue criterion in 2D cyclic loading

C. Lu*, J. Melendez, J.M. Martínez-Esnaola

CEIT-IK4 and TECNUN (University of Navarra), P. Manuel Lardizabal 15, 20018 San Sebastián, Spain

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ABSTRACT

In this article, a universally applicable multiaxial fatigue criterion in 2D cyclic loading is proposed, which can be used for a great variety of materials and loading conditions. A strain-based fatigue parameter is defined and, at the same time, a new failure model is proposed to overcome the weaknesses of other mechanisms used previously. In addition, the influence of non-proportional loading, maximum, minimum and mean loading, the influence of both normal and shear components, Poisson effect, different failure types, etc. can also be taken into consideration. A huge number of materials and loading conditions are used to validate the capabilities of the proposed methodology. The results show that the new multiaxial fatigue criterion provides excellent life predictions for all the materials and loading conditions used in this work. The proposed approach can be regarded as a universally applicable multiaxial fatigue criterion in 2D cyclic loading.

1. Introduction

Fatigue is a common problem in engineering. Uniaxial fatigue is reasonably well described, due to the simple state of loading. However, multi-axial fatigue still remains a problem under discussion, because there are many factors that affect the material's response. These include the variation of the principal stress directions [1,2], the stress ratio [3], the mean stress [4,5], the non-proportionality of loading components [6,7], the phase angle [8–10], etc.

Many multiaxial fatigue criteria have been proposed by different researchers. Generally, these criteria can be divided into stress-based methods, strain-based methods, criteria based on the combination of both stress and strain, and energy-based criteria. Generally speaking, stress-based criteria are used in high cycle fatigue, because plasticity is restrained in this regime, even though there are always plastic strains at the micro-scale. Strain-based criteria are commonly used in low cycle fatigue, as plastic deformation is observable at the macro-scale (these criteria can also be used in high cycle fatigue). Some researchers argue that using only stress or strain components is not sufficient to capture material hardening. This is the reason why criteria based on the combination of both stress and strain components or energetic magnitudes have been also proposed. Some of the most widely applied methods are briefly reviewed below.

Wang and Brown [11] put forward a strain-based criterion (WB criterion); the specific form of their fatigue parameter (FP) is given by

$$\mathrm{FP} = \frac{\Delta\gamma_{max}}{2} + S\varepsilon_n^* \tag{1.1}$$

where $\Delta\gamma_{max}$ is the maximum range of the shear strain on the critical plane, ε_n^* is the normal strain excursion on the same plane, i.e., the difference between the relative maximum and minimum values of the normal strain within one cycle, and S is a material-dependent constant. The critical plane is defined as the plane where the range of shear strain is maximum. In this criterion, both shear and normal components contribute to the fatigue parameter. However, the influence of the mean normal strain (and therefore, the different effect of the tensile and compressive normal strains) is not accounted for. In addition, according to the study before [1], the fatigue failure plane is mainly affected by the maximum principal stress or strain. However, it is assumed here that the maximum shear strain range plane is the critical plane.

Fatemi and Socie [12] propose a fatigue criterion based on stress and strain magnitudes (FS criterion). Although earlier in time, it can be regarded as a modification of the WB criterion where the normal strain range is replaced by the maximum normal stress,

$$\mathrm{FP} = \gamma_{max}(1 + n\sigma_n^{max}/\sigma_y) \tag{1.2}$$

where γ_{max} is the maximum value of shear strain on the critical plane, σ_n^{max} is the maximum normal stress on that plane, σ_y is yield stress and n is a material-dependent constant. The critical plane in this method is the plane subjected to the maximum shear strain during the loading process. This criterion assumes that the shear strain is the key factor in the fatigue damage and that the maximum normal stress contributes to accelerate the fatigue damage. In the FS criterion, the different influence of the tensile and compressive normal components are accounted for. However, in the special cases where the maximum normal stress is zero (i.e., the normal stress is always compressive), this criterion

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* Corresponding author at: Parque Tecnológico de San Sebastián Paseo Mikeletegi, N° 48 20009, Donostia – San Sebastián, Spain.
E-mail addresses: clu@ceit.es, se7en_luchun@163.com (C. Lu).

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ignores the influence of the magnitude of the compressive component.

Smith et al. [13] use an energy-type magnitude to formulate fatigue damage (SWT criterion). They assume that the maximum normal strain energy can be a proper parameter to correlate with fatigue life. The SWT criterion is given by

$$\mathrm{FP} = \sigma_{max}\varepsilon_a \tag{1.3}$$

where σ_{max} is the maximum normal stress during the loading cycle and ε_a is the normal strain amplitude acting on the same plane. The critical plane in this case is that corresponding to the maximum principal stress. The SWT criterion takes into account, in an indirect manner, the influence of the mean stress through the combination of $\sigma_{\max}$ and ε_{a}. But only stress and strain components related to the normal strain energy are used in the FP. However, it does not take into account the fact that the shear strain energy also influences the fatigue damage.

Chen et al. [14] advocate considering the influence of both normal and shear strain energies (CXH criterion). In addition, they distinguish between tension-type failure and shear-type failure. For the tension-type failure mechanism, the fatigue parameter is described by

$$\mathrm{FP} = \Delta\varepsilon_1^{max}\Delta\sigma_1 + \Delta\gamma_1\Delta\tau_1 \tag{1.4}$$

where $\Delta\varepsilon_1^{max}$ is the maximum range of the normal strain on the critical plane, $\Delta\sigma_1$ is the corresponding range of the normal stress, and $\Delta\gamma_1$ and $\Delta\tau_1$ are the ranges of shear strain and shear stress, respectively, on the same plane. The critical plane is the one at which the range of the normal strain reaches the maximum value. For a shear-type failure, the CXH fatigue parameter changes to

$$\mathrm{FP} = \Delta\gamma_{max}\Delta\tau + \Delta\varepsilon_n\Delta\sigma_n \tag{1.5}$$

where $\Delta\gamma_{max}$ is the maximum range of the shear strain on the critical plane, $\Delta\tau$ is the corresponding range of the shear stress, and $\Delta\varepsilon_n$ and $\Delta\sigma_n$ are the ranges of the normal strain and normal stress, respectively, on the same plane. The critical plane is defined as the one corresponding to the maximum range of shear strain. In the CXH criterion, the progress is that both normal and shear strain energies are used to establish the fatigue parameter. But the decision on how to choose between tension-type and shear-type failure is not based on physical arguments. In fact, the recommendation in [14] is to use the most conservative of the predictions obtained with the two assumptions. In addition, only the ranges of the normal and shear stress and strain components are used in the definition of fatigue damage; therefore, the different influence of the tension and compression magnitudes cannot be distinguished.

Varvani-Farahani [15] proposes an energy-based criterion (Varvani criterion) that accounts for the influence of the mean stress with the additional advantage that there is no fitting coefficient (apart from material-dependent coefficients with physical meaning). The normal and shear energy components are weighted by the axial and shear fatigue properties in the form

$$\mathrm{FP} = \frac{1}{\sigma_f'\varepsilon_f'}(\Delta\sigma_n\Delta\varepsilon_n) + \frac{1+\sigma_n^m/\sigma_f'}{\tau_f'\gamma_f'}\left(\Delta\tau_{max}\Delta\frac{\gamma_{max}}{2}\right) \tag{1.6}$$

where σ_f' and ε_f' are the axial fatigue strength and axial fatigue ductility coefficients, τ_f' and γ_f' are the shear fatigue strength and shear fatigue ductility coefficients. $\Delta\sigma_n$ and $\Delta\varepsilon_n$ are the ranges of the normal stress and normal strain on the critical plane. $\Delta\tau_{max}$ and $\Delta\frac{\gamma_{max}}{2}$ are the maximum ranges of the shear stress and shear strain, and σ_n^m is the mean normal stress on the critical plane, which can be calculated by the average value of the maximum and minimum value of the normal stress on that plane. The critical plane in this criterion is defined as the one at which the shear strain range is maximum. At the same time, the influence of mean stress and normal and shear strain energies is taken into consideration. However, there are many material properties that, in practical engineering applications, are not available in many cases.

The review above indicates that there is not a definite criterion that can be used for all materials and loading conditions. The purpose of this work is to establish a multiaxial fatigue criterion that can be applied to all materials and loading conditions. This is what we mean by the term "universally applicable criterion" in 2D cyclic loading. In the following, a strain-based fatigue parameter is proposed and a new failure mechanism is advocated to overcome the weaknesses of other approaches. The influence of non-proportional loading, maximum, minimum and mean stresses, normal and shear components, Poisson effect, different failure types, etc. can also be taken into consideration. A huge number of materials and loading conditions are used to validate the capabilities of the proposed multiaxial fatigue criterion. The results show that the new multiaxial fatigue criterion provide an excellent life prediction for all the materials and loading paths used in this article. We suggest that the proposed multiaxial fatigue criterion can be regarded as method of general applicability in 2D cyclic loading.

2. Fatigue failure mechanism

Several failure mechanisms have been identified and described in the literature. In the tension-type failure mechanisms, it is assumed that the normal stress-strain components are the main factors controlling the fatigue process. On the other hand, in the shear-type failure mechanisms, the shear stress-strain components are considered to be the relevant magnitudes to describe fatigue damage. However, only the normal or the shear components alone are not sufficient to predict the fatigue damage [14]; it is necessary to consider the interaction of both types of magnitudes [16]. For example, Socie and Marquis [17,18] have proposed a shear failure mechanism controlled by the maximum shear (stress-strain) range with the corresponding normal components contributing to accelerate or slow down fatigue damage, as illustrated in Fig. 1.

In the failure mechanism postulated in [17,18], it is assumed that the failure plane is that corresponding to the maximum shear strain. The failure process is the interaction of both normal and shear components of stress and strain: the tensile components on the failure plane help to open the crack, which will accelerate the fatigue process; on the contrary, the compressive normal components on the failure plane tend to close the crack, which will slow down the fatigue process. At first glance, this failure mechanism can explain the fatigue process reasonably well. However, it has to be pointed out that this failure mechanism is only operative when a crack already exists. In other words, for the fatigue initiation stage, this failure mechanism loses its rationality, as there is no a physical crack.

2.1. Proposed failure mechanism

In order to overcome the weak points of the failure mechanisms discussed above, a new failure mechanism is proposed in this work, which is based on the material deformation. It is well recognized that the orientation of the failure plane is not random, but it rather follows a preferential plane [1]. It is reasonable to admit that the deformation perpendicular to the failure plane controls the fatigue damage. We can

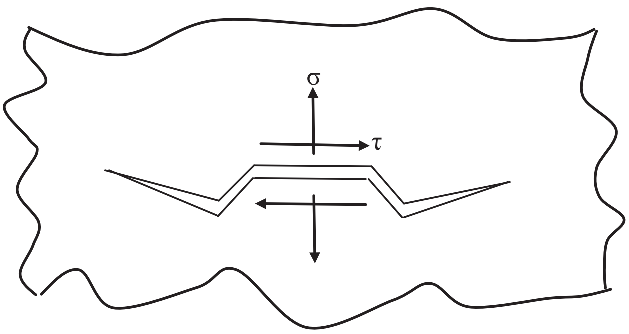

Fig. 1. Cracking mechanism under combined normal and shear components [17].