International Journal of Fatigue 82 (2016) 572–578

Contents lists available at ScienceDirect

International Journal of Fatigue

journal homepage: www.elsevier.com/locate/ijfatigue

The influence of stress state on the exponent in the power law equation of fatigue crack growth

E. Amsterdam *, F. Grooteman

National Aerospace laboratory (NLR), Voorsterweg 31, 8316 PR Marknesse, The Netherlands

ARTICLE INFO

Article history:
Received 19 February 2015
Received in revised form 1 September 2015
Accepted 10 September 2015
Available online 21 September 2015

Keywords:
Power law
Fatigue crack growth rate
Aluminium
Pivot point
Stress state

ABSTRACT

Paris et al. were the first to show that the fatigue crack growth rate in metals shows a power law relationship with the stress intensity factor range. It is generally accepted that the exponent is material-dependent. However, it is also true that the empirical Paris equation is dimensionally correct only when the dimensions of the constant in the equation are changed with the power law exponent. In the present paper it will be shown that for 29 identical fatigue crack growth tests on aluminium alloy 7075-T7351 the exponent changes between specimens and crack lengths. Fractography and the crack length measurements show that the exponent is higher and varies at crack lengths where crack growth is dominated by plane strain conditions. The power law exponent decreases and is similar for all specimens after the transition to plane stress conditions at higher crack lengths. The mathematical concept of a pivot point is used to model crack growth with two different exponents using a dimensionally correct equation. It also allows modelling the crack growth variation in all specimens by varying only one parameter, the power law exponent for the plane strain condition.

1. Introduction

1.1. Fatigue crack growth rate equations

Paris et al. were the first to show that the fatigue crack growth rate (FCGR) has a power law relationship with the stress intensity factor range, ΔK [1]:

$$\frac{da}{dN} = C_R \cdot \Delta K^n \tag{1}$$

where da/dN is the FCGR, C_R is a constant and n is the power law exponent. C_R and n are both regarded as material parameters. The stress intensity factor range is defined as:

$$\Delta K = \beta(a)(S_{\max} - S_{\min})\sqrt{\pi a} \tag{2}$$

where S is the applied stress and a is the crack length. The geometry correction factor $\beta(a)$ depends on the geometry and the crack length. From 1963, many adaptations of Eq. (1) (the Paris equation) have been proposed to incorporate the experimental results at low ΔK, near the supposed threshold, and at high ΔK, near the fracture toughness regime. Also the effective stress intensity factor range, ΔK_{eff}, was introduced to account for different FCGRs at different stress ratios, R:

$$\Delta K_{eff} = f(R) \cdot \Delta K \tag{3}$$

where $f(R)$ is a function of the stress ratio, R:

$$R = \frac{S_{\min}}{S_{\max}} \tag{4}$$

$$\frac{da}{dN} = C \cdot \Delta K_{eff}^n \tag{5}$$

Different correction functions, $f(R)$, have been proposed, e.g. by Elber, Newman, and Schijve, and they all attributed the correction function entirely to crack closure [2–7]. However, Alderliesten recently showed that the correction function can be regarded mostly as a correction for energy and for a small part for crack closure that occurs at low stresses [8]. This also indicates that the fundamentals of fatigue crack growth are not so well understood as one would expect after more than 50 years of extensive research after introduction of the Paris equation [1].

Linear elastic fracture mechanics (LEFM) has not been able to deduce the Paris equation from basic principles or a physical model, and so it is still an empirical equation. Another major issue is the fact that this equation is dimensionally correct only when the dimensions of the constant C_R are changed with a change in the exponent, n. In the past many people have introduced various crack growth rate equations that can be grouped together by the following general formulation [8–10]:

* Corresponding author. Tel.: +31 88 511 4284.
E-mail address: emiel.amsterdam@nlr.nl (E. Amsterdam).

http://dx.doi.org/10.1016/j.ijfatigue.2015.09.013
0142-1123/© 2015 Elsevier Ltd. All rights reserved.

$$\frac{da}{dN} \propto \sigma^l a^m \quad (6)$$

In the present investigation the stress amplitude has been held constant during the test and equal for all specimens. Therefore the only change in crack growth rate results from a change in crack length. In the case of Eq. (5), m is equal to $n/2$ and l is equal to n and therefore $2m$:

$$\frac{da}{dN} \propto \Delta S^n \sqrt{a}^n = \Delta S^{2m} a^m \quad (7)$$

1.2. Pivot point

It is generally accepted that the exponent is material-dependent. Zuidema et al. also observed a decrease in the exponent for aluminium alloy AA 2024 when shear lips develop [11–13]. Usually, fatigue cracks start flat and grow in mode I (tensile mode), until at a certain ΔK_{eff} the specimen (or component) starts to develop shear lips. The shear lip development in aluminium alloys AA 2024-T3 and AA 7075-T6 starts at a critical growth rate of about $1 \cdot 10^{-7}$ m/cycle [14]. The shear lip thickness increases with increasing ΔK_{eff}, but it also depends on the cycle frequency and environment: for constant ΔK_{eff} fatigue tests the shear lip thickness is constant and proportional to the logarithm of the test frequency [14]; a more aggressive environment decreases the shear lip thickness [14].

Development of shear lips implies that fatigue crack growth occurs in mixed mode, I + II + III [14]. However, conventionally the mode I ΔK_{eff} is then still used for FCGR modelling. In the present investigation this will also be the case. Zuidema et al. indicate that the shear lips are not responsible for a decrease in n, but that they, like n, are an effect from an underlying phenomenon [14].

When crack growth rate data are equation-fitted, then a change in n also gives rise to a change in the constant C. Many people have reported a linear relationship between *log C* and the exponent, n [15–29];

$$\log C = -zn + b \quad (8)$$

where constant z is the slope and constant b the intercept on the ordinate axis. This indicates that the constant C is not independent of the exponent n. Taking the logarithm of Eq. (5) results in;

$$\log C = -\log(\Delta K_{eff})n + \log\left(\frac{da}{dN}\right) \quad (9)$$

If there is a linear relationship between *log C* and n, the constants z and b result in particular values of ΔK_{eff} and da/dN:

$$\Delta K_{eff,z} = 10^z \quad (10)$$

and

$$\frac{da}{dN_b} = 10^b \quad (11)$$

Combining Eqs. (10) and (11) with Eq. (8) results in the following expression for the constant C:

$$C = \frac{da}{dN_b} \Delta K_{eff,z}^{-n} \quad (12)$$

Substituting Eq. (12) for the constant in Eq. (5) results in a relative crack growth rate equation that is dimensionally correct:

$$\frac{da}{dN} = \frac{da}{dN_b}\left(\frac{\Delta K_{eff}}{\Delta K_{eff,z}}\right)^n \quad (13)$$

When $\Delta K_{eff,z}$ is equal to 1 Eq. (13) is equal to Eq. (5). The linear relationship between *log C* and the exponent, n, introduces a so-called pivot point in the da/dN vs. ΔK_{eff} plot. The pivot point is located at the coordinates $\Delta K_{eff,z}$, da/dN_b and allows describing the variation between specimens tested under similar conditions by only a variation in n.

A number of papers have been written about the pivot point. However, there is much doubt about there being a physical basis for a pivot point [24–28]. Kohout states that it is a mathematical consequence of the fitting process, whereby a pivot point represents only the centre of gravity of the data points [28]. Reviewing several papers for different materials [23–25], lost concludes that the pivot point is material-dependent and may be associated with a transition mechanism in the FCGR curve [25].

In the present paper it will be shown that n not only changes between materials and specimens, but also with crack length: n is higher and varies at crack lengths where crack growth is dominated by plane strain conditions. After the transition to plane stress conditions, at greater crack lengths, n decreases and is similar for all specimens. The change in n causes a pivot point transition in the FCGR curve. Therefore, the mathematical concept of a pivot point will be used to model crack growth with two different exponents using a dimensionally correct equation. This allows modelling the crack growth variation in all specimens by varying only one parameter, the power law exponent n derived from the plane strain condition.

2. Experimental set-up and results

Constant amplitude crack growth tests were performed on 29 middle tension specimens according to ASTM E647–00. The specimen dimensions were 500 mm × 160 mm × 6.35 mm, and were obtained from 6.35 mm thick aluminium alloy AA7075-T7351 plate material. The rolling direction was in the length of the specimens. Fatigue crack growth starter notches were central holes (1.6 mm diameter) with 0.7 mm deep electric discharge machined slots on either side of the hole (total starter notch length of 3 mm).

Holes were drilled at 8 mm above and below the start notch hole for copper pins. These were used for automated crack length measurements by direct current potential drop (DCPD). The current was introduced to the specimen at the specimen clamping. The area next to the starter notch was polished for optical crack growth measurements on both front and rear sides of the specimens. The optical measurements (4 for each crack length measurement) were averaged and used to check the DCPD measurements.

A constant amplitude 13.5 Hz sinusoidal load was introduced by an electrohydraulic test machine with a 200 kN load cell. The maximum and minimum stresses during the load cycles were 80 MPa and 8 MPa, corresponding to a stress ratio $R = 0.1$. The specimens were pre-cracked to a crack length of 2 mm (4 mm in total). Failure was defined as occurring at 40.8 mm (single side) even though actual failure occurred at higher crack lengths. The failure criterion of 40.8 mm was used because all specimens gave crack length data points up to at least 40.8 mm. For higher crack lengths, a single crack length could not always be measured with the DCPD-system, owing to very high crack growth rates.

Fig. 1 shows examples of the obtained fatigue crack growth curves: specimen 1 had the lowest number of cycles to failure and specimen 26 one of the highest number of cycles to failure. In total, the number of cycles to failure ranged between 103,150 and 133,350 cycles, see Fig. 2.

3. Discussion

It was noticed that the number of cycles to grow the crack from about 10 mm to 40.8 mm was approximately the same for all specimens (see Fig. 3) and that the shapes of the curves were similar