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Review and application of Rainflow residue processing techniques for accurate fatigue damage estimation

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ABSTRACT

Most fatigue loaded structural components are subjected to variable amplitude loads which must be processed into a form that is compatible with design life calculations. Rainflow counting allows individual stress cycles to be identified where they form a closed stress–strain hysteresis loop within a random signal, but inevitably leaves a residue of open data points which must be post-processed. Comparison is made between conventional methods of processing the residue data points, which may be non-conservative, and a more versatile method, presented by Amzallag et al. (1994), which allows transition cycles to be processed accurately.

This paper presents an analytical proof of the method presented by Amzallag et al. The impact of residue processing on fatigue calculations is demonstrated through the application and comparison of the different techniques in two case studies using long term, high resolution data sets. The most significance is found when the load process results in a slowly varying mean stress which is not fully accounted for by traditional Rainflow counting methods.

1. Introduction

The calculation of conservative load cycle spectra is a fundamental aspect of fatigue design, requiring an estimate to be made of expected operational loading conditions. Complex lifecycle loading may be simplified by dividing the process into discrete load cases, such as take-off and steady flight conditions for the analysis of aircraft components. Fatigue life can then be quantified in terms of time to crack initiation through the concept of linear damage accumulation, or by the application of crack growth models. Both approaches utilise information about the range, mean and number of stress cycles that will occur [1].

The identification of individual fatigue loading cycles within a random stress amplitude time series is achieved through the use of a suitable cycle counting algorithm. Typical methods include level-crossing counting, range-pair counting, reservoir counting, and Rainflow counting. Variations of these algorithms are included in the ASTM cycle counting standard [2].

1.1. Background to the Rainflow counting algorithm

Rainflow (RF) counting has become the most widely accepted method for the processing of random signals for fatigue analysis, and testing has demonstrated good agreement with measured fatigue lives when compared to other counting algorithms [3]. The concept was first developed by Matsuishi and Endo [4], where the identification of cycles was likened to the path taken by rain running down a pagoda roof. In the paper, the authors defined a full RF cycle as a stress range formed by two points which are bounded within adjacent points of higher and lower magnitude; as the stress path returns past the first turning point it can be seen to form a cycle as described by a closed stress–strain hysteresis loop (Fig. 1a). For the case where successive stress points are either converging or diverging, the hysteresis curves do not form a closed loop (Fig. 1b). For this case the authors assumed that fatigue damage could be attributed to each successive range as half-cycles.

The method was further developed by Okamura et al. [5] and Downing and Socie [6] as a vector based algorithm which identified full RF cycles and half-cycles based on a three-point criteria without the need to rearrange the data series, and enabled efficient utilisation in computer software. This greatly reduced the data

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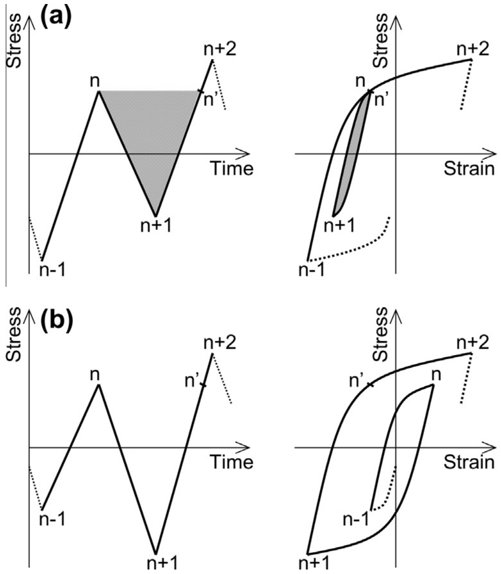

Fig. 1. (a) Stress time series of turning points and the corresponding closed stress–strain hysteresis loop formed by points n, $n+1$ and n'. (b) Diverging stress time series and the corresponding open stress–strain hysteresis curves.

storage requirements as the stress signal could be read into the algorithm in real-time and processed directly into RF cycle spectra. This definition of the algorithm has been refined and included in the ASTM cycle counting standard [2]. Amzallag et al. [7] conducted a wide ranging industry consultation and defined a standardised algorithm which identified RF cycles based on a four-point criterion. The three and four point versions of the algorithm were shown to identify the same cycles by McInnes and Meehan [8], who presented a series of fundamental properties of RF counting to demonstrate the equivalence of the two methods. Although various forms of the RF algorithm exist, the four-point algorithm presents the most unambiguous criterion for the identification of closed hysteresis loops, and is defined below.

1.2. Four-point Rainflow counting criterion

RF counting requires the time history to be first processed into a Peak-Valley (PV) series consisting of local maxima and minima which define the turning points, or load reversals, of a time series. Point x_m is identified as a local maxima or minima within a time series of length M if,

$$x_{m-1} < x_m > x_{m+1} \text{ or } x_{m-1} > x_m < x_{m+1} \quad m = 2, 3, 4, \ldots, M-1 \tag{1}$$

Once the data have been filtered according to the PV criteria, full RF cycles are identified in the range formed by points x_n to x_{n+1} if they meet the four-point criterion,

$$|x_{n-1} - x_n| \geqslant |x_n - x_{n+1}| \leqslant |x_{n+1} - x_{n+2}| \quad n = 2, 3, 4, \ldots, N-2 \tag{2}$$

where N signifies the length of the PV filtered series. If the range formed by points x_n to x_{n+1} meets the four-point criterion then the points are recorded before deleting them from the PV series, thus enabling further ranges to be formed between the adjacent points x_{n-1} and x_{n+2}. The process is repeated until all ranges which meet the four-point criterion are recorded and deleted from the PV series.

Storage of the counted ranges is achieved with a two dimensional histogram to record the cycle stresses. The form of the histogram may be chosen to preserve detailed cycle hysteresis information which may be significant in further statistical analysis, for example with the min–max or max–min matrices where cycles are binned according to the loading sequence [9]. As a minimum, the histogram should record the cycle range and mean stress levels as inputs to final damage calculations.

1.3. Rainflow residue

Once all full RF cycles which meet the four-point criterion have been identified and deleted from the PV series, a 'residue' of data points will typically remain. The residue consists of a series of diverging data points from the start to the maximum and minimum points, followed by a converging section of points to the end of the PV data series. Referring to Fig. 2, no remaining closed hysteresis cycles can be identified within a diverging or converging series as no further ranges are bounded by adjacent points of higher and lower value. However, as the stress path formed by the residue constitutes some of the largest ranges in the original series, they should be accounted for if a conservative estimate of fatigue damage is to be made. Two dominant methods exist in the literature to process the RF residue and are outlined in Sections 2.1 and 2.2.

Whenever a subset of a longer time history is RF counted, cycle ranges which are formed between points which span beyond the subset have the potential to be cropped. If there is a large variation in the mean stress level, which is not fully contained within the subset period, then some of the largest cycles will not be accounted for. These cycles are termed "transition cycles" or "ground cycles" [10], and a degree of artificiality will be introduced if the residue data points are processed as an isolated set, as closed hysteresis cycles cannot be formed. The only way to accurately identify all RF cycles within a data set according to the four-point criterion is to process the entire time history consecutively, yet the application of RF counting algorithms must always utilise a finite length of data, as chosen by the analyst and by limitations on computational capacity.

Glinka and Kam [11] presented an approach which allowed extended time periods to be read and processed incrementally, thus limiting the required computational capacity by minimising the amount of data required to be handled by the RF algorithm at any one time. A more versatile method is included in Amzallag et al. [7, pp. 292–293] which addresses the same issue by concatenating consecutive residue periods which remain after RF processing. However, although the method allows transition cycles to be accounted for accurately according the four-point criterion, it has not found widespread acknowledgement and no generalised proof of the methodology has been presented.

The three methods of processing the RF residue periods are presented in Section 2 below. An analytical proof is presented in Section 3 which demonstrates the equivalence of cycles which are identified from the residue concatenation methodology outlined in [7] with those which would be identified by RF processing a continuous series. In Section 4, the different approaches are

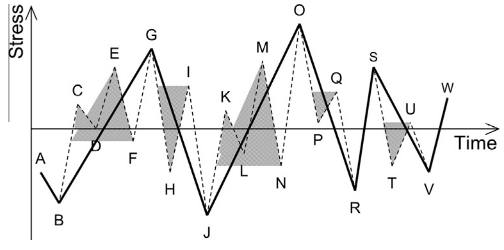

Fig. 2. Residue remaining after application of the four-point criterion (points connected by solid line). Full RF cycles would be identified between points *C–D*, *E–F*, *H–I*, *K–L*, *M–N*, *P–Q*, *T–U*.