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Microplane damage model for fatigue of quasibrittle materials: Sub-critical crack growth, lifetime and residual strength

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ABSTRACT

In contrast to metals and fine grained ceramics, fatigue in concrete and other quasibrittle materials occurs in a large fracture process zone that is not negligible compared to the structure size. This causes the fatigue to be combined with triaxial softening damage whose localization is governed by a finite material characteristic length. A realistic model applicable to both has apparently not yet been developed and is the goal of this paper. Microplane model M7, shown previously to capture well the nonlinear triaxial behavior of concrete under a great variety of loadings paths, is extended by incorporating a new law for hysteresis and fatigue degradation, which is formulated as a function of the length of the path of the inelastic volumetric strain in the strain space. The crack band model, whose band width represents a material characteristic length preventing spurious localization, is used to simulate propagation of the fatigue fracture process zone. Thus the fatigue crack with its wide and long process zone is simulated as a damage band of a finite width. For constant amplitude cycles, the model is shown to reproduce well, up to several thousands of cycles, the Paris law behavior with a high exponent previously identified for concrete and ceramics, but with a crack growth rate depending on the structure size. Good agreement with the crack growth histories and lifetimes previously measured on three-point bend beams of normal and high strength concretes is demonstrated. The calculated compliance evolution of the specimens also matches the previous experiments. The model can be applied to load cycles of varying amplitude, to residual strength under sudden overload and damage under nonproportional strain tensor variation. Application to size effect in fatigue is relegated to a follow-up paper, while a cycle-jump algorithm for extrapolation high-cycle fatigue with millions of cycles remains to be researched.

1. Introduction and objective

Although extensive research results on the sub-critical fatigue crack growth in metals and ceramics have been accumulated they are not applicable to quasibrittle materials, for three reasons:

(1) The modeling of fatigue is inseparable from the modeling of distributed triaxial softening damage, which is rather intricate for quasibrittle materials;
(2) because of material heterogeneity, the size of the fracture process zone (FPZ) is not negligible compared to structural dimensions (see Fig. 1); and
(3) the finiteness of FPZ size, representing the material characteristic length, engenders a structural size effect on the rate of fatigue fracture growth (the size effect will be addressed in a follow-up article).

The size effect on the Paris law for fatigue fracture growth has been demonstrated in two previous studies and described on the basis of equivalent linear elastic fracture mechanics (LEFM) [1,2]. But no general model for combined fatigue and nonlinear triaxial softening damage appears to exist. The purpose of this paper is to develop such a model. The microplane model M7, which is a new and most advanced model for damage in concrete, with the broadest experimental verification, is adopted as the starting point.

Quasibrittle materials are heterogenous materials with brittle constituents. They include concretes, as the archetypical case, fiber composites, tough ceramics, sea ice, rock, stiff soils, rigid foams, wood, coal, bone, various bio- and bio-inspired materials, etc. All brittle materials become quasibrittle on sufficiently small scale. Thus brittle materials in micrometer scale devices (MEMS) must be expected to exhibit quasibrittle fatigue behavior similar to the behavior of concrete on the scale of meters.

The crack growth in brittle materials follows the Paris law [3], which reads,

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0142-1123/

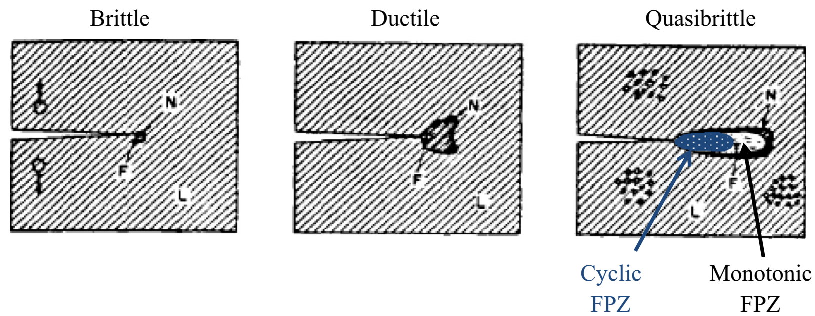

Fig. 1. Shape of nonlinear zone and fracture process zones [34] (L = linear zone, F = Fracture process zone, N = nonlinear zone).

$$\frac{da}{dN} = C(\Delta K)^m \tag{1}$$

where C and m are empirical constants depending on factors such as the environment, and load ratio [4]. The Paris law has been shown to apply also to concrete if the structure size is kept constant [1,2], and under this constraint it probably applies to all quasibrittle materials. The Paris law exponent m is typically between 2 and 4 for metals, while for concrete and ceramics it is about 10–50 [1,2,5,6]. Unlike metals, concrete exhibits a size effect on the crack growth rate [1,2], and so do rocks [7]. The microstructural fatigue mechanism, which is well known for metals [8,9], is not yet well understood for concrete but is sure to be quite different. Its understanding will be helped by the FPZ analysis which follows.

Fracture in concrete is characterized by distributed post-peak softening damage within a FPZ whose size is not negligible compared to structural dimensions [10]. The difference from embrittled metals is that the FPZ that travels forward with the fatigue crack growth is large. For metals, the FPZ is of micrometer size, and thus is negligible for specimens of centimeter size or larger. Therefore, interactions between triaxial damage and fatigue crack growth are insignificant. Not so, however, for quasi-brittle materials. The load cycles produce residual stresses in the cyclic FPZ, which is expected to be smaller than the monotonic FPZ but still large enough for such interactions. Under stable subcritical crack growth, the fatigue crack grows while interacting with progressive damage in the advancing FPZ. In the final stage, as the effective stress intensity factor of the equivalent LEFM crack increases, the crack advance accelerates and leads to stability loss, i.e., failure. The FPZ undergoing softening damage in concrete has been modeled by the cohesive crack model, in which the full FPZ width is lumped into an interface between two opposite crack faces and the softening damage is described by a traction-separation law [11–13]. In this approach, however, the tensorial behavior of the FPZ is not captured and the minimum possible spacing of parallel non-localizing cracks is not enforced.

To capture full tensorial behavior in the FPZ, including the effect of normal stresses parallel to the crack plane, one may use the crack band model (see [14], with implementation in [15] for a commercial code and arbitrary constitutive law). In this model, the inelastic softening damage is smeared within a crack band of a finite reference width that is a material property. The band width can be changed by adjusting the steepness of post-peak softening so as to maintain the same energy dissipation (although this is not what is done here, in order to minimize the approximation error associated with changing the crack band width). The reference band width serves as a material characteristic length representing a localization limiter, which prevents spurious mesh sensitivity of results [16]. For a very thin crack band, the cohesive crack model can be approximated closely. Another advantage of the crack band model is that it can simulate the width of the FPZ, and consequently it can represent with correct spacing a system of parallel cracks when they do not localize, as in the presence of reinforcing bars or for some thermal stress distributions.

1.1. Previous studies of cohesive and constitutive models for fatigue

To predict the fatigue crack growth, the unload–reload hysteresis of the material must be captured realistically. Especially, the deformation increment over a cycle must be predicted correctly. The hysteresis was simulated by a number of researchers using the cohesive crack model.

In [17], the uniaxial cohesive stress–strain curve was adjusted downward after each cycle by an amount corresponding to the energy dissipated in the cycle. Another model [18] introduced a hysteretic cohesive law with different incremental stiffnesses depending on whether the cohesive surface opens or closes. This model was able to reproduce fatigue crack growth of Paris-law type and also some deviations from this law including short crack growth and the retarding effect of sudden overload in metals. In [19], an extension of this model with development of residual tensile stresses in the FPZ was shown to simulate fatigue crack growth under cyclic compression. In [20], a cohesive zone model with polynomial expressions for loading and unloading paths was proposed. Other similar approaches included the models by Horii [21], Bahn and Su [22], Bažant and Planas [10] and the continuous function model by Hordjik and Reinhardt [23]. Some studies also proposed for fatigue continuum damage models which, however, were based on rather simple or simplified tensorial damage behavior [24–27].

None of the aforementioned models are able to predict the Paris law exponent or the size effect. They all aim to describe the fatigue crack growth as a separate phenomenon and cannot realistically model concrete damage and failure in general situations where fatigue may be combined with general nonlinear triaxial behavior of concrete. To develop such a model is the objective of this article.

2. Formulation of constitutive model for fatigue damage

2.1. Overview of microplane model M7

Similar to Taylor models for polycrystalline metals [28–31], the basic idea of the microplane constitutive models, due to Taylor [32], is to express the constitutive law not in terms of tensors but in terms of the stress and strain vectors acting on generic plane of arbitrary orientation within the material, which is called the *microplane*. The use of vectors is conceptually simpler. It makes possible a semi-intuitive use of physical concepts such as frictional slip, microcrack opening and compression splitting (in the classical tensorial approach, the internal friction, for example, is represented by a relation between the first and second invariants even though it occurs only on planes of distinct orientation). Initially