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Modified shear lag theory based fatigue crack growth life prediction model for short-fiber reinforced metal matrix composites

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ABSTRACT

Closed form expressions for the low cycle and high cycle fatigue crack growth lives have been derived for the randomly-planar oriented short-fiber reinforced metal matrix composites under the total strain-controlled conditions. The modeling was based on fatigue-fracture mechanics theory under both the small scale and the large scale yielding conditions. The modified shear lag theory was considered to describe the effect of yielding strength. The present model is essentially a crack growth model because crack initiation period in short fiber reinforced metal matrix composite is much shorter; hence, not assumed to play a dominant role in the calculation of fatigue crack growth life. The effects of short-fiber volume fraction (V_f), cyclic strain hardening exponent (n') and cyclic strain hardening coefficient (K') on the fatigue crack propagation life are analyzed for aluminum based SFMMCs at different levels of cyclic plastic strain values. It is observed that the influence of fatigue crack growth resistance increases with increase in cyclic strain hardening exponent (n') and decreases when volume fraction (V_f) or cyclic strain hardening coefficient (K') increases. The present MSL theory based fatigue crack growth life prediction model is an alternative of modified rule of mixture and strengthening factor models. The predicted fatigue life for SFMMC shows good agreement with the experimental data for the low cycle and high cycle fatigue applications.

1. Introduction

The inadequacy of metals and alloys in providing higher strength, stiffness and fatigue life to structures led to the development of particulate, short-fiber and continuous fiber-reinforced composite materials [1–3]. There are several studies to understand the strain-controlled fatigue damage tolerance characteristics of discontinuous reinforced Metal Matrix Composite (MMC) [4–8]. Surprisingly, crack growth based fatigue life prediction modeling for the Short-Fiber Reinforced Metal Matrix Composite (SFMMC) received much less attention. One of the reasons could be the heterogeneity posed by the random-planar oriented SFMMCs accounting for the difficulties in fatigue life calculations. Over the last two decades, limited analytical Fatigue Crack Growth Life Prediction (FCGLP) models for SFMMCs are available in literature [4,5,9]. Begley and McMeeking [9] proposed the analytical FCGLP model for fiber-reinforced MMCs where crack growth was calculated by reducing the crack tip stress intensity range.

In metals, there is a Cyclic Plastic Zone (CPZ) ahead of crack tip under the cyclic loading, where the fatigue crack growth is primarily controlled by localized plastic deformation [10–12]. Large numbers of theories were proposed to predict the size of CPZ [13–16]. One of them is based on Linear Elastic Fracture Mechanics (LEFM), where Rice [13] proposed the fatigue crack propagation theory using stress intensity factor at the crack tip under Small Scale Yielding (SSY) condition. On contrary, Dugdale [14] calculated CPZ size based on Elastic Plastic Fracture Mechanics (EPFM) theory when large plastic deformation occurred ahead the crack tip, i.e., Large Scale Yielding (LSY) condition. Further, Lu and Chow [15] modified this model by incorporating the crack tip cyclic large plasticity. Zhang and Du [16] performed an elastic–plastic Finite Element Analysis (FEA) of crack under both the SSY and LSY conditions, displaying that the fatigue crack growth rate was dominated

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Abbreviations: ΔCTOD, crack tip opening displacement; CPZ, cyclic plastic zone; EPFM, elastic plastic fracture mechanics; FCGLP, fatigue crack growth life prediction; FDZ, fatigue damage zone; FEA, finite element analysis; HCF, high cycle fatigue; LCF, low cycle fatigue; LEFM, linear elastic fracture mechanics; LSY, large scale yielding; MMC, metal matrix composite; MROM, modified rule of mixture; MSL, modified shear lag; SF, strengthening factor; SFMMC, short fiber metal matrix composite; SSY, small scale yielding.

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Nomenclature

a	crack length
a_i	initial crack length
a_f	final crack length
C_ε	constraint
E_m	Young's modulus of matrix
E_f	Young's modulus of fiber
s	length-to-diameter (aspect) ratio of short-fiber
N_f	fatigue crack growth life
N_{fs}	fatigue crack growth life under SSY
N_{fl}	fatigue crack growth life under LSY
V_f	short-fiber volume fraction
w_{CPZ}	cyclic plastic zone size for metals
$w_{CPZ,C}$	cyclic plastic zone size for SFMMC
$w_{FDZ,C}$	fatigue damage zone size for SFMMC
Y	crack geometry correction factor
n'	cyclic strain hardening exponent
K'	cyclic strength coefficient
$\frac{da}{dN}$	fatigue crack growth rate
ΔCTOD	crack tip opening displacement
ΔJ	cyclic J integral
$\frac{\Delta K_{eff}}{2}$	effective stress intensity range
$\frac{\Delta \sigma_{eff}}{2}$	effective stress amplitude
$\frac{\Delta \sigma}{2}$	cyclic stress amplitude
$\frac{\Delta \varepsilon_{pl}}{2}$	cyclic plastic strain amplitude
σ_y	cyclic yielding strength of metal
σ_{cy}	cyclic yielding strength of SFMMC
σ_{my}	cyclic yielding strength of matrix
σ_{um}	ultimate tensile strength of matrix material
λ_s	correction factor under the SSY
λ_l	correction factor under the LSY
λ_{MSL}	correction factor for MSL model
λ_{MROM}	correction factor for MROM model
λ_{SF}	correction factor for SF model
ω	numerical factor

by the plasticity in front of the crack tip. Later, Ding et al. [17,18] extended the concept of CPZ to SFMMCs solely based on crack propagation mechanism from microstructural features, where they proposed Low Cycle Fatigue (LCF) life model for short-fiber reinforced aluminum alloy MMC based on Modified Rule of Mixture (MROM) [17]. Subsequent work on LCF model was based on Strengthening Factor (SF) theory for pure aluminum based SFMMC [18]. Ding et al. [19] used LEFM approach for deriving the LCF life expression for particle-reinforced MMC based on shear lag theory to take the effect of yield strength.

In the present work, FCGLP model for randomly-planar oriented SFMMCs has been developed by incorporating both the yielding conditions, i.e., SSY using LEFM approach and LSY using EPFM approach. The model incorporates crack-tip cyclic plastic deformation as well as cyclic crack growth kinetics. To bear the load of hard reinforcements, Modified Shear Lag (MSL) strengthening theory was implemented, keeping the effect of cyclic yield strength into the consideration. The effects of short-fiber volume fraction, cyclic strain hardening exponent and cyclic strength coefficient on the fatigue crack propagation life of SFMMC at different levels of cyclic plastic strain were investigated. The analytical results are compared with two other analytical models proposed by Ding et al. [17,18] and validated with published experimental results [8,17,18]. The developed generalized model could be applicable for LCF and High Cycle Fatigue (HCF) applications.

2. Fatigue crack growth life prediction model for SFMMCs

The present FCGLP model for randomly-planar oriented SFMMCs is developed by incorporating both the yielding conditions, i.e., SSY and LSY, and does not consider the material microstructure variability, fibers specific orientation or variation of fiber length of MMCs in model. Initially, expression for CPZ size was derived using MSL strengthening theory [20] under both the SSY and LSY conditions. The CPZ was then used for determining the size of Fatigue Damage Zone (FDZ). The actual degradation process in SFMMC takes place within the crack tip FDZ, the area very close to the crack tip, under the SSY. Subsequently, the local driving force (cyclic J integral) was calculated using the interaction energy between FDZ and crack tip stress–strain field. The cyclic J integral is directly related to the range of Crack Tip Opening Displacement (ΔCTOD) which is equal to one half of the crack extension for each cycle, i.e., fatigue crack growth rate [11]. Finally, LCF and HCF life expressions were derived.

In literature, the strengthening effect of reinforced fiber on the matrix is attributed to two factors, i.e. one is to strengthen the microstructure of the matrix and other is to constrain the plastic flow of the matrix. Jiang et al. [20] proposed a modified shear lag model, which ascribes the consideration of stress concentration in the fiber end and stress variation of matrix behavior from elastic state to the plastic state to calculate the fiber stress in plastic region, to relate the yield strength of short fiber-reinforced MMCs with the yield strength of the matrix alloy.

The cyclic yielding strength for short-fiber composites [20] may be expressed as

$$\sigma_{cy} = \left[1 + \frac{8V_f^2 s^2 (E_f - E_m)}{3(E_f + 4V_f s^2 E_m)}\right]\sigma_{my} \tag{1}$$

where σ_{cy} is cyclic yielding strength of composite, σ_{my} is cyclic yielding strength of matrix, V_f is short-fiber volume fraction, s is the length-to-diameter (aspect) ratio of short-fiber, E_m and E_f are Young's moduli of matrix and fiber materials.

2.1. FCGLP model incorporating small scale yielding

Fig. 1 indicates the local shearing in FDZ during crack growth very near to the crack tip and inside the CPZ. The CPZ size under the SSY can be expressed as [19,21,22]

$$w_{CPZ,C} = \lambda_s \left(\frac{\Delta K_{eff}}{2\sigma_{my}}\right)^2 \tag{2}$$

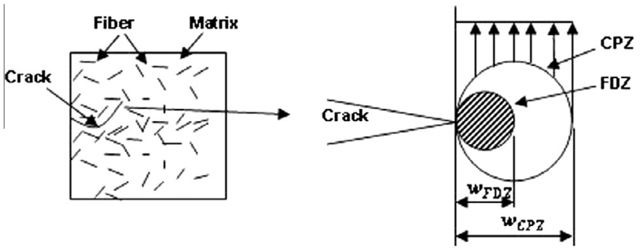

Fig. 1. The cyclic plastic zone and fatigue damage zone ahead of fatigue crack-tip region in SFMMCs.