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Entropic characterization of metal fatigue with stress concentration

M. Liakat, M.M. Khonsari *

Department of Mechanical Engineering, Louisiana State University, Baton Rouge, LA 70803, USA

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ABSTRACT

The accumulation of thermodynamic entropy generation in a fatigue degradation process, beginning with a pristine specimen and ending at fracture, is referred to as the fatigue fracture entropy (FFE). In this paper, the concept of FFE is applied to study the effect of stress concentration on metal fatigue. Experiments involve uniaxial tension–compression fatigue tests carried out with solid cylindrical un-notched and V-notched specimens made of both medium-carbon steel 1045 and aluminum 6061. Finite element simulations are also performed with both types of specimens to study their thermal response under cyclic load and to predict the corresponding FFE. Experimental and theoretical results show that the FFE of these materials are nearly constant for each type of the specimens. Under the conditions tested the presence of stress concentration is shown to reduce the amount of hysteresis energy generation in the specimen gage section compared to the un-notched specimen under the identical loading conditions. Also shown is that the FFE decreases significantly due to the presence of stress concentration. An empirical correlation is proposed that can predict the fatigue life of a V-notched specimen based on the hysteresis energy per cycle and the FFE of an un-notched specimen. Predicted and experimental fatigue lives of V-notched specimens are found to be in good agreement.

1. Introduction

Many approaches are available for analyzing fatigue degradation such as strain-energy method [1–4], stress-life method [4], and strain-life method [4–6]. Among these approaches, strain-energy method utilizes cyclic plastic strain energy (also known as the hysteresis energy) generated by the irreversible plastic deformation in a material as an index of fatigue characterization. Most of this energy is converted into heat and dissipates to the surroundings [3,7,8].

Fatigue is an example of dissipated process wherein the accumulation of disorder is inherently associated with generation of thermodynamic entropy in accordance with the second law of thermodynamics [9–14]. Specifically, the amount of entropy generation in a fatigue degradation process is found to be a useful parameter for the assessment of material damage in different processes such as cyclic mechanical loading [7], thermomigration, electromigration, and thermomechanical loading [15–17]. Research shows that the accumulation of thermodynamic entropy—starting with a pristine specimen without stress concentration and ending at the fatigue fracture—is a material property called *fatigue fracture entropy* (FFE), which is useful for estimation of fatigue life. Experiments show that within the range of operating conditions tested FFE is independent of the amplitude and the frequency of load, the geometry of the specimen, and the type of fatigue load [10,18–21]. The concept of entropy accumulation has been successfully applied to monitor the evolution of fatigue damage in specimens without stress concentration [22] and in steam turbine rotors [23] subject to both constant and variable amplitude loading.

The thermodynamic entropy generation and temperature evolution depend on the plastic strain energy dissipation as the fatigue degradation progresses. It therefore follows that the presence of stress concentration in a component, which is known to cause significant changes in the stress–strain distribution, must affect the rate of thermodynamic entropy generation and temperature evolution. Hence, the application of the concept of FFE to characterize the effect of stress concentration in a notched specimen calls for further study.

This paper is devoted to entropic characterization of the fatigue behavior of solid cylindrical un-notched and V-notched specimens made of two different metals. The effect of stress concentration on the evolution of entropy, the shape and size of hysteresis loops, and the FFE are discussed. Numerical simulations are conducted using the finite element method (FEM) to simulate the thermal response of both types of specimens under cyclic load and to determine FFE due to cyclic stress. An empirical correlation

* Corresponding author. Tel.: +1 2255789192; fax: +1 2255785924.
E-mail address: khonsari@me.lsu.edu (M.M. Khonsari).

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Nomenclature

$\boldsymbol{A_k}$	thermodynamic forces associated with internal variables
C	specific heat (W m^{-1} K^{-1})
e	specific internal energy (J kg^{-1})
E	modulus of elasticity (GPa)
f	frequency (Hz)
h	heat transfer coefficient (W m^{-2} K^{-1})
k	thermal conductivity (W m^{-1} K^{-1})
k_a	thermal conductivity of air (W m^{-1} K^{-1})
K_f	fatigue notch factor
K_t	stress concentration factor
L	effective length (m)
L_c	characteristic length of specimen (m)
L_R	load ratio
n	surface normal parameter
N	number of load cycle/consumed life
N_f	fatigue life (cycle)
N_{fv}	fatigue life of V-notched specimen (cycle)
N_{fu}	fatigue life of un-notched specimen (cycle)
N_{PR}	Prandtl number
$\boldsymbol{q}$	heat flux (W m^{-2})
R	radius (mm)
R_a	arithmetic average of surface roughness (μm)
s	specific entropy (J Kg^{-1} K^{-1})
S_{ev}	fatigue limit of V-notched specimen (MPa)
S_{eu}	fatigue limit of un-notched specimen (MPa)
t_f	fatigue life (s)
T	temperature (K)
T_a	ambient temperature (K)
T_v	temperature of V-notched specimen (K) at the notch-root
T_u	temperature of un-notched specimen (K) in the middle of gage section
U	velocity of air (m s^{-1})
$\boldsymbol{V_i}$	internal variables
W_p	hysteresis energy per second (MJ m^{-3} s^{-1})
ΔW_p	hysteresis energy per cycle (MJ m^{-3} cycle^{-1})
ΔW_{pv}	hysteresis energy per cycle of V-notched specimen (MJ m^{-3} cycle^{-1})
ΔW_{pu}	hysteresis energy per cycle of un-notched specimen (MJ m^{-3} cycle^{-1})
γ	fatigue fracture entropy (MJ m^{-3} K^{-1})
γ_v	FFE of V-notched specimen (MJ m^{-3} K^{-1})
γ_u	FFE of un-notched specimen (MJ m^{-3} K^{-1})
$\Delta\varepsilon$	strain range
$\boldsymbol{\varepsilon^e}$	elastic strain tensor
$\boldsymbol{\dot{\varepsilon}^p}$	time rate of plastic strain tensor (s^{-1})
ε_0	surface emissivity
ν	kinematic viscosity of air (m^2 s^{-1})
ρ	density (kg m^{-3})
$\boldsymbol{\sigma}$	stress tensor
σ	nominal stress (MPa)
$\Delta\sigma$	stress range (MPa)
σ_{max}	maximum stress due to stress concentration (MPa)
σ_0	Stephan–Boltzmann constant (W m^{-2} K^{-4})
$\boldsymbol{\Psi}$	specific free energy (J kg^{-1})

is proposed that can predict the fatigue life of a V-notched specimen based on the hysteresis energy per cycle and FFE of an un-notched specimen subject to identical loading and environmental conditions.

The outline of the paper is as follows. In Section 2, theoretical analysis for the calculation of the FFE is described followed by the materials, experimental procedure, instrumentation, and experimental results in Section 3. Section 4 presents the finite element model, the boundary conditions, and the validation of the FEM analysis and the predicted FFE results. The development of a correlation for the fatigue life prediction of a V-notched specimen and its validation are discussed in Section 5 followed by the conclusions.

2. Theory

The first law of thermodynamics postulates that in the absence of internal heat source, conservation of energy principle can be expressed as [14]:

$$\rho\dot{e} = \boldsymbol{\sigma} : \dot{\boldsymbol{\varepsilon}} - div\ \boldsymbol{q} \tag{1}$$

where ρ is the density, $\dot{e} = \dot{\boldsymbol{\Psi}} + T\dot{s} + \dot{T}s$ stands for the time rate of specific internal energy, $\boldsymbol{\Psi}$ is the specific free energy, T is the absolute temperature, s signifies the specific entropy, $\boldsymbol{\sigma}$ and $\dot{\boldsymbol{\varepsilon}}$ denote the stress tensor and the time rate of strain tensor, respectively, and $\boldsymbol{q}$ is the heat flux. $\boldsymbol{\Psi}$ is a function of T, elastic strain, $\boldsymbol{\varepsilon}_e$, and internal variables associated with microstructure, V_i (where i is the number of internal variables), and its derivative with respect to time yields $\dot{\boldsymbol{\Psi}} = \frac{\partial \boldsymbol{\Psi}}{\partial \boldsymbol{\varepsilon}^e} : \dot{\boldsymbol{\varepsilon}}^e + \frac{\partial \boldsymbol{\Psi}}{\partial T}\dot{T} + \frac{\partial \boldsymbol{\Psi}}{\partial V_i}\dot{V}_i$.

Substituting the value of $\dot{\boldsymbol{e}}$ and $\dot{\boldsymbol{\Psi}}$ into Eq. (1) gives:

$$\boldsymbol{\sigma} : \dot{\boldsymbol{\varepsilon}} - div\ \boldsymbol{q} = \rho\left[\frac{\partial \boldsymbol{\Psi}}{\partial \boldsymbol{\varepsilon}^e} : \dot{\boldsymbol{\varepsilon}}^e + \frac{\partial \boldsymbol{\Psi}}{\partial T}\dot{T} + \frac{\partial \boldsymbol{\Psi}}{\partial \boldsymbol{V}_i}\dot{\boldsymbol{V}}_i + T\dot{s} + \dot{T}s\right] \tag{2}$$

The laws of thermoelasticity state that $\sigma = \rho\frac{\partial \boldsymbol{\Psi}}{\partial \boldsymbol{\varepsilon}^e}$ and $s = -\frac{\partial \boldsymbol{\Psi}}{\partial T}$ and thermodynamic forces associated with internal variables $\boldsymbol{A}_i = \rho\frac{\partial \boldsymbol{\Psi}}{\partial \boldsymbol{V}_i}$. Fourier's law states that $\boldsymbol{q} = -k\ grad\ T$ where k is the thermal conductivity. The time rate of entropy generation $\dot{s} = -\frac{1}{\rho}\frac{\partial \boldsymbol{\sigma}}{\partial T} : \dot{\boldsymbol{\varepsilon}}^e + \frac{\partial s}{\partial T}\dot{T} - \frac{1}{\rho}\frac{\partial \boldsymbol{A}_i}{\partial T}\dot{\boldsymbol{V}}_i$, specific heat $C = T\frac{\partial s}{\partial T}$, and total strain rate $\dot{\boldsymbol{\varepsilon}} = \dot{\boldsymbol{\varepsilon}}^e + \dot{\boldsymbol{\varepsilon}}^p$. Substituting these into Eq. (2) leads to [14]:

$$k\nabla^2 T = \rho C\dot{T} - \boldsymbol{\sigma} : \dot{\boldsymbol{\varepsilon}}^p + \boldsymbol{A}_i\dot{\boldsymbol{V}}_i - T\left(\frac{\partial \boldsymbol{\sigma}}{\partial T} : \dot{\boldsymbol{\varepsilon}}^e + \frac{\partial \boldsymbol{A}_i}{\partial T}\dot{\boldsymbol{V}}_i\right) \tag{3}$$

Since elastic strain energy, $\boldsymbol{\sigma} : \dot{\boldsymbol{\varepsilon}}^e$, is reversible and does not contribute to the heat generation, this term is neglected in Eq. (3).

The second law of thermodynamics postulates that the time rate of volumetric entropy production, $\dot{\gamma}$, of a deformed body is always greater than or equal to the time rate of heating divided by the temperature which is expressed by the Clausius–Duhem inequality reads [14]:

$$\dot{\gamma} = \frac{\boldsymbol{\sigma} : \dot{\boldsymbol{\varepsilon}}^p}{T} - \frac{\dot{\boldsymbol{q}}}{T^2}\cdot grad\ T - \frac{\boldsymbol{A}_i\dot{\boldsymbol{V}}_i}{T} \geq 0 \tag{4}$$

Research shows that $\boldsymbol{A}_i\dot{\boldsymbol{V}}_i$ represents only 5–10% of the entropy generation due to $\boldsymbol{\sigma} : \dot{\boldsymbol{\varepsilon}}^p$ and thermo-mechanical coupling effect, $\frac{\partial \boldsymbol{\sigma}}{\partial T} : \dot{\boldsymbol{\varepsilon}}^e$, is negligible compared to the mean temperature rise [14]. Hence, it is assumed that $\boldsymbol{A}_i\dot{\boldsymbol{V}}_i \approx 0$, $\frac{\partial \boldsymbol{\sigma}}{\partial T}\dot{\boldsymbol{\varepsilon}}^e \approx 0$, and $\frac{\partial \boldsymbol{A}_i}{\partial T}\dot{\boldsymbol{V}}_i \approx 0$. Thus, Eqs. (3) and (4) simplify as, respectively [19]:

$$k\nabla^2 T = \rho C\dot{T} - \dot{W}_p \tag{5}$$

$$\dot{\gamma} = \frac{\dot{W}_p}{T} - \frac{\dot{\boldsymbol{q}}}{T^2}\cdot grad\ T \tag{6}$$