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# Bayesian model selection and parameter estimation for fatigue damage progression models in composites

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ABSTRACT

A Bayesian approach is presented for selecting the most probable model class among a set of damage mechanics models for fatigue damage progression in composites. Candidate models, that are first parameterized through a Global Sensitivity Analysis, are ranked based on estimated probabilities that measure the extent of agreement of their predictions with observed data. A case study is presented using multi-scale fatigue damage data from a cross-ply carbon–epoxy laminate. The results show that, for this case, the most probable model class among the competing candidates is the one that involves the simplest damage mechanics. The principle of Ockham's razor seems to hold true for the composite materials investigated here since the data-fit of more complex models is penalized, as they extract more information from the data.



## 1. Introduction

Modeling the progression of fatigue damage in fiber-reinforced polymer (FRP) composite materials is still a challenging problem with important implications for safety and cost in a wide range of engineering applications. Although composites are high-performance materials with high strength-weight ratios, they are susceptible to fatigue degradation from the beginning of lifespan [1]. Unlike metals, fatigue in composites is governed by complex multi-scale damage processes driven by several internal fracture events that ultimately lead to the alteration of the macro-scale mechanical properties [1,2]. The inherent complexity of this process implies uncertainty in modeling, that not only includes the uncertainty in model parameters but also the uncertainty arising from the choice of a particular *model class* (e.g., the parameterized mathematical structure of the model for predicting damage behavior). This paper focuses on quantifying the model uncertainty of a set of candidate damage mechanics models for composites, through a full Bayesian approach that simultaneously estimates the plausibility of each individual model class along with the uncertainty of the underlying model parameters.

Some researchers have started investigating the role of uncertainties in modeling the behavior of composites materials. For example, Sriramula and Chryssanthopoulos [3] discussed different stochastic modeling approaches for analyzing uncertainties at the ply-level, coupon-level and component-level. Uncertainty quantification methods have also been used to assess the uncertainty in the material properties [4,5] and extended to study a variety of phenomena such as elastic response [6,7], aeroelastic behavior [8,9], and failure [10,11], among others. While the topic of uncertainty quantification is receiving increased attention in the composites literature, there is still an evident need for a rigorous treatment of the uncertainty in modeling the progression of fatigue damage of composite materials.

In particular, the Bayesian approach has been successfully applied for uncertainty quantification in fatigue, but mainly in the context of metals [12–19]. For example, Cross et al. [13] and Sankararaman et al. [14] used Bayesian inference to estimate parameters underlying crack growth behavior. Sankararaman et al. [16] used dynamic Bayesian networks for model parameter estimation and calculated Bayes factors to quantify model uncertainty. Recently, Chiachío et al. [20] have proposed a stochastic model for damage evolution and used a Bayesian model selection framework to account for model uncertainty. Therefore it seems reasonable to explore the applicability of these methods to fatigue damage modeling in composites materials, where the benefits of the Bayesian approach can be fully exploited due to the inherent complexity and the existence of multiple competing models.

This paper proposes a rigorous Bayesian framework to account for modeling uncertainty in application to the problem of fatigue

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damage progression in composite materials. To this end, Bayes' Theorem is applied at two levels: first, to quantify the uncertainty regarding the model parameters for a specific model class, and second, to assess the probability of each model class within a set of candidate damage mechanics models. Here, probability is interpreted as a multi-valued logic that expresses the degree of belief of a proposition conditioned on the given information [21,22]. Consequently, the approach has the advantage of being able to quantify the uncertainty associated with (1) model parameters and (2) model choice for the damage behavior, and then to further make predictions of fatigue degradation that rigorously incorporate different types of modeling uncertainty in a quantitative manner.

A set of five model classes pertaining to three families of damage mechanics models [23] (i.e., shear-lag [24,25], variational [26] and crack opening displacement (COD) [27,28]) is chosen to represent the relation between the macro-scale stiffness reduction and the micro-scale damage, due to matrix-cracks. Damage mechanics models are preferred over other analytical approaches (e.g. continuum damage mechanics models or synergistic damage mechanics models [23]) due to their efficiency in relation to the assumptions adopted, and for being well connected with the physics of the underlying damage process. Moreover, these models have the ability to adapt to different systems (materials, loading conditions, etc.) without much training and furthermore, they can incorporate monitoring data in the Structural Health Monitoring (SHM) context.

Each candidate damage mechanics model is subsequently embedded into the *modified Paris' law* [29], that is used in this study to model the propagation forward in time of the matrix-cracks density. This two-level modeling approach results in a large number of uncertain parameters, leading to a computationally-intensive inference problem. To reduce the dimensionality of the problem without significantly altering the underlying uncertainty in the model output, a model input tuning is carried out by means of a Global Sensitivity Analysis (GSA) [30]. This allows to determine in advance the subset of parameters that are most "sensitive" to the model output uncertainty.

As a validation example, the proposed Bayesian framework is applied to damage data for matrix-crack density and stiffness reduction from a tension–tension fatigue experiment performed over a cross-ply CFRP laminate [31,32]. The results show that more complicated damage mechanics models not only involve more complicated analysis and adjustable parameters, but also do not yield higher probabilities in explaining the observed damage response. In this context, the *evidence* (also called as *marginal likelihood*) of each model class is revealed as a suitable measure to know the overall ability of the candidate model to predict the observed damage response, avoiding the extremes of over-fitting or under-fitting the data.

The paper is organized as follows. Section 2 discusses the theory behind fatigue damage in composites and presents the proposed methodology for fatigue damage modeling. In Section 3, the Bayesian inference framework for both model parameters and model classes is presented together with the problem of model parameterization using GSA. Section 4 is devoted to providing implementation details for conferring computational efficiency to the Bayesian inference problem. In particular, the computation of the likelihood function using the Graphics Processing Unit (GPU) is illustrated. In Section 5, the proposed framework is applied to a set of fatigue damage data to serve as an example. Finally Section 6 discusses the results and Section 7 provides concluding remarks.

## 2. Fatigue damage modeling

Typically, fatigue damage is perceived as a progressive or sudden change of the macro-scale mechanical properties, such as stiffness or strength, as a consequence of different fracture modes that evolve at the micro-scale along the lifespan of the structure [2]. In this research work, longitudinal stiffness loss is chosen as the macro-scale damage variable. In contrast to the strength, the stiffness can be measured through non-destructive methods during operation, which is of key importance for the model-updating approach proposed. At the micro-scale level, matrix micro-cracking [33] is selected as the dominant fracture mode for the early stage of damage accumulation.

Matrix cracks usually initiate from internal defects in 90° plies during first loading cycles, and grow rapidly along fibers direction spanning the entire width of the specimen [33]. Continued loading leads to formation of new cracks between the already formed cracks thereby progressively increasing the matrix-cracks density of the ply until saturation. This saturated state, usually termed as *characteristic damage state* [1], is long recognized as a precursor of more severe fracture modes in adjacent plies, such as delamination and fiber breakage [34,35], which may subsequently lead to the catastrophic failure of the laminate. In addition, matrix micro-cracking may itself constitute failure of the design when micro-cracks induced degradation in properties exceeds the predefined threshold.

To accurately represent the relation between this micro-scale damage mode and its manifestation through macro-scale properties, several families of *micro-damage mechanics* models can be found in the literature [23]. These models, that are grounded on first principles of admissible ply stress fields in presence of damage, can be roughly classified into (1) computational methods, (2) semi-analytical methods and (3) analytical methods. Among them, computational and semi-analytical methods have been shown to be promising approaches, however they are computationally intensive; hence a large number of repeated evaluations in a simulation-based inference procedure is computationally prohibitive. Therefore, we focus here on the set of analytical models to address the relationship between stiffness loss and micro-crack density. Three types of analytical models are considered: *shear-lag* models [24,25], *variational* models [26], and *crack opening displacement* based models [27,28].

Shear-lag models use one-dimensional approximations of the equilibrium stress field after cracking to derive expressions for stiffness properties of the cracked laminate. The main modeling assumption is basically that, in the position of matrix cracks, axial load is transferred to uncracked plies by the axial shear stresses at the interfaces. These models have received the most attention in the literature and, as a consequence, a vast number of modifications and extensions can be found. However, as stated by Talreja and Singh [23], all the one-dimensional shear-lag models are virtually identical, except for the choice of the *shear-lag parameter*, as explained later in this section.

Variational models are based on a two-dimensional approximation of the equilibrium stress field, that in contrast to shear-lag analysis, is obtained from the Principle of Minimum Complementary Energy [36,37].

Finally, COD-based models use a 3-D homogenization procedure derived from the study of the average crack-face opening displacement of a single matrix crack as a function of the applied load, that can be calculated either analytically [27] or numerically [38,39,28]. While shear-lag and variational models are mostly applicable to cross-ply laminates (i.e. those with stacking sequence $\left[0_{\frac{n_0}{2}}/90_{n_{90}}/0_{\frac{n_0}{2}}\right]$, where $n_{0,90}$ = total number of plies at 0° and 90°, respectively), COD-based models are applicable to general laminates with an arbitrary distribution of matrix cracks. The reader is referred to the recent work of Talreja and Singh [23] for a detailed overview of these models, but for the sake of clarity, the key formulation is appropriately reproduced here with a uniform notation.