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A generalized fatigue damage parameter for multiaxial fatigue life prediction under proportional and non-proportional loadings

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ABSTRACT

In this paper, two different forms of an original multiaxial fatigue damage parameter related to the maximum fatigue damage plane are proposed for performing fatigue life prediction under various loading conditions loadings. The proposed fatigue damage parameters have been applied to uniaxial and multiaxial loading conditions for geometrically different bodies. Both the damage parameters are correlated to sets of experimental data published in the literature to verify the prediction accuracy of the damage parameters. The damage parameter in the form of the GSA, when applied to the uniaxial loading, provides very good correlations with four sets of experimental mean stress fatigue data for Incoloy 901 super alloy, ASTM A723 steel, 7075-T561 aluminum alloy and 1045 HRC 55 steel. In the case of multiaxial loadings, both the GSE and GSA parameters are found to correlate well with fatigue data of 1045 steel and Inconel 718 tubular specimens under proportional and non-proportional loadings. In addition, the damage parameters show reasonably acceptable correlations with experimental fatigue data of SAE 1045 steel notched shafts subjected to proportional and non-proportional loadings.

1. Introduction

It is quite common that engineering components experience multiaxial fatigue failure, since most machine components are subjected to multiaxial load conditions in service and multiaxial fatigue generally results from the geometry and external loading. Multiaxial fatigue analysis is a very complex process in comparison to uniaxial fatigue. Different from the uniaxial fatigue problem, the multiaxial fatigue involves complex stress and strain states, load histories and fatigue damage parameters relating the fatigue life. In recent decades, a large number of research studies have been conducted to develop a successful multiaxial fatigue damage parameter. Although numerous damage parameters have been proposed during the past decades to predict the multiaxial fatigue failure, most of them are limited to specific load cases and material and there is no universally accepted damage parameter yet. In general, most of the damage parameters can be classified as stress-based, strain-based and energy-based damage parameters. Since there are several good review papers of existing multiaxial fatigue damage parameters can be found in literature [1–3], these damage parameters are not reviewed again in this paper. However, some important feature of the stress-based, strain-based and energy-based damage parameters are summarized here. The stress-based damage parameters [4–9] are generally used for infinite and/or high cycle fatigue regime where the plastic deformation is negligible. The successful application of a particular stress-based damage parameter depends on material coefficients determined from experiments. Strain-based fatigue damage approaches [10–14] for multiaxial fatigue have generally successful applications in low cycle fatigue regime where plasticity deformation is significant. However, Strain-based damage approaches can be applied to both low and high cycle fatigue regimes since strain-based and stress-based approaches merge in high cycle fatigue. Strain-based critical plane fatigue damage parameters have developed from experimental observation that a fatigue crack nucleates and grows along either shear planes or tensile planes depending on the material, stress and strain states. Energy-based damage parameters [15–23] came from the basis that strain energy may be considered as a fatigue damage quantity of a material, can be formulated as the product of strain and stress components. As mentioned previously, this paper is not intended to be a review paper since the review of existing fatigue damage parameters can be found in the literature in great detail. Recently, Fatemi and Shamsaei [36] presented a very good overview paper of some important issues in multiaxial fatigue. The multiaxial fatigue life prediction remains a challenging problem due to its general wide practical applications, therefore additional research studies should be still required for accurate and reliable multiaxial fatigue assessment. Therefore, two different forms of an original fatigue damage parameter, which are based on the critical plane approach,

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are proposed in this paper to improve some of the shortcomings of the current fatigue damage parameters.

2. Multiaxial fatigue damage parameter

The successful design of machine components subjected to complex multiaxial loadings requires that an effective multiaxial fatigue damage parameter be available, which can accurately estimate the fatigue life of those components under complex states of stresses and strains. It is well known that the local strain approach has been well adapted as a practical engineering method in fatigue assessment of mechanical components. A fatigue damage parameter, which quantifies the fatigue damage as a function of certain stress and strain variables such as normal strain and maximum stress, relates the fatigue damage to fatigue life cycles. In past few decades, a significant number of fatigue damage parameters have been developed [2,6,10,11,17] and no universal consensus has been reached on the best approach to multiaxial fatigue problem. However, the critical plane concept has been considered as one of the most successful methods in multiaxial fatigue applications and critical plane-based damage parameters have gained general popularity due to their reasonably accurate life prediction capabilities. Most of the critical plane-based fatigue damage parameters are formulated in the form of stress and strain components or a combination of stresses and strains associated with the critical plane. However, these damage parameters have limitations taking into account mean stress effects, non-proportional hardening, and requirement for additional material constants to characterize the fatigue damage. In general, a successful multiaxial fatigue damage parameter should include following important features.

- Simple, efficient and applicable to a variety of fatigue loading conditions e.g., uniaxial loadings and multiaxial loadings including proportional and non-proportional loadings.
- Applicable to low and high cycle fatigue regimes.
- Includes mean stress effects in the fatigue damage parameter.
- Reflects the constitutive behavior of material and the non-proportional hardening.
- Physically consistent with the continuum mechanics.
- Defined without using any additional material coefficient.
- Load path dependent.
- Damage mechanism process.

Considering these important features, a fatigue damage parameter is needed to accurately simulate the multiaxial fatigue behavior of materials. Therefore, two different forms of an original fatigue damage parameter incorporating some of these important features have been proposed for multiaxial fatigue life estimations of different geometrical specimens (tubular and notched solid specimens) under a various loading conditions. The proposed damage parameters are based on the maximum fatigue damage plane rather than the maximum normal or shear strain plane. Both stress and strain terms are included in the proposed damage parameters in order to incorporate the material deformation response in the damage parameter.

2.1. Generalized strain energy (GSE) fatigue damage parameter

In consideration of these important features desired for a successful fatigue damage parameter and to overcome the shortcomings of the existing damage parameters, an original fatigue damage parameter(s) based on generalized strain energy (GSE) that considers the specific plane(s) experiencing the maximum amount of fatigue damage is proposed. The GSE damage parameter includes the normal and shear strain energy terms on a plane experiencing the maximum generalized strain energy, can be expressed in the following equation:

$$W^*_{gen} = \left(\tau_{\max}\frac{\Delta\gamma^e}{2} + \frac{\Delta\tau}{2}\frac{\Delta\gamma^p}{2} + \sigma_{n,\max}\frac{\Delta\varepsilon^e_n}{2} + \frac{\Delta\sigma_n}{2}\frac{\Delta\varepsilon^p_n}{2}\right)_{\max} = f(N_f) \tag{1}$$

A schematic representation of the GSE damage parameter is shown in Fig. 1. As seen from this figure, the GSE damage parameter includes the sum of elastic shear strain energy $\left(\tau_{\max}\frac{\Delta\gamma^e}{2}\right)$, plastic shear strain energy $\left(\frac{\Delta\tau}{2}\frac{\Delta\gamma^p}{2}\right)$, elastic normal strain energy $\left(\sigma_{n,\max}\frac{\Delta\varepsilon^e_n}{2}\right)$ and plastic normal strain energy $\left(\frac{\Delta\sigma_n}{2}\frac{\Delta\varepsilon^p_n}{2}\right)$ terms. The GSE damage parameter accounts for effects of the mean stress and the non-proportional hardening through the elastic shear and normal strain energy terms $\left(\tau_{\max}\frac{\Delta\gamma^e}{2}\right)$ and $\left(\sigma_{n,\max}\frac{\Delta\varepsilon^e_n}{2}\right)$ by including the maximum shear stress, $\tau_{\max}$ and maximum normal stress, $\sigma_{n,\max}$ components in the formulation of the damage parameter. Contrary to the strain based fatigue damage parameters, the proposed strain energy parameter, W^*_{gen} is acceptable from the continuum mechanics viewpoint that energy components are mathematically consistent and can be added algebraically. The GSE damage parameter can be related to the elastic and plastic strain energy density contributed by the normal and shear stresses and strains on the critical plane. The shear strain energy terms reflect the initiation and growth of cracks, and the normal strain energy terms accelerate the crack growth. Similar to the Chu parameter [17,24], the proposed GSE damage parameter is based on a plane experiencing the maximum damage parameter i.e. the average contribution from tensile and shear energy terms rather than the plane of maximum shear or normal strains.

2.2. Generalized strain amplitude (GSA) fatigue damage parameter

The shear strain energy terms in Eq. (1) can be normalized with the shear stress amplitude, $\Delta\tau/2$ and the normal strain energy terms can be normalized with the normal stress amplitude, $\Delta\sigma_n/2$ to transform the generalized strain energy parameter, Eq. (1) to the form of generalized strain amplitude (GSA), Eq. (2).

The proposed fatigue damage parameter in the form of GSA can be written as:

$$\frac{\Delta\varepsilon^*_{gen}}{2} = \left(\frac{\tau_{\max}}{\Delta\tau/2}\frac{\Delta\gamma^e}{2} + \frac{\Delta\gamma^p}{2} + \frac{\sigma_{n,\max}}{\Delta\sigma_n/2}\frac{\Delta\varepsilon^e_n}{2} + \frac{\Delta\varepsilon^p_n}{2}\right)_{\max} = f\ (N_f) \tag{2}$$

Based on Basquin's equation [25] the shear stress amplitude, $\Delta\tau/2$ and the normal stress amplitude, $\Delta\sigma_n/2$ for uniaxial stress state, can be expressed as:

$$\begin{aligned}\tfrac{\Delta\tau}{2} &= \tau'_f(2N_f)^{b\gamma}\\ \tfrac{\Delta\sigma_n}{2} &= \sigma_a = \sigma'_f(2N_f)^b\end{aligned} \tag{3}$$

In the case of $2N_f = 1$, the shear and normal stress amplitude, Eq. (3) can be given:

$$\begin{aligned}\tfrac{\Delta\tau}{2} &= \tau'_f\\ \tfrac{\Delta\sigma_n}{2} &= \sigma'_f\end{aligned} \tag{4}$$

Substituting the shear and normal stresses from Eq. (4) into Eq. (2), the multiaxial fatigue damage parameter based on the generalized strain amplitude can be expressed as:

$$\frac{\Delta\varepsilon^*_{gen}}{2} = \left(\frac{\tau_{\max}}{\tau'_f}\frac{\Delta\gamma^e}{2} + \frac{\Delta\gamma^p}{2} + \frac{\sigma_{n,\max}}{\sigma'_f}\frac{\Delta\varepsilon^e_n}{2} + \frac{\Delta\varepsilon^p_n}{2}\right)_{\max} = f\ (N_f) \tag{5}$$