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Microstructure-sensitive weighted probability approach for modeling surface to bulk transition of high cycle fatigue failures dominated by primary inclusions

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ABSTRACT

A simulation-based probabilistic strategy is developed to characterize the surface to bulk transition of high cycle fatigue failures dominated by primary inclusions. The probability of fatigue crack initiation in the surface region is calculated by computing the expected number of critical fatigue hot spots in this region. This is done by considering the probability of inclusion-matrix debonding and the fatigue crack initiation potency of partially-debonded inclusions for a given load ratio and stress amplitude.

A case study is presented whereby the surface initiation probability is studied in uniaxial strain-controlled cyclic loading simulations of round smooth specimens of the fine grained powder metallurgy (PM) processed Ni-base superalloy IN100. The fatigue crack initiation potency of partially-debonded non-metallic inclusions is assessed by calculating the Fatemi–Socie (FS) critical plane parameter from generalized plain strain crystal plasticity finite element simulations. Idealized spherical ceramic inclusions with homogeneous linear elastic isotropic material properties are considered to isolate the FS parameter sensitivity to inclusions' size, stress amplitude, and polycrystalline microstructure realization around the inclusion.

1. Introduction

The mechanical alloying and casting processes used to make polycrystalline metallic materials often introduce undesirable nonmetallic inclusions and pores that are large relative to the mean grain size. These are often the dominant fatigue damage sites at the low stress amplitudes that correspond to the high cycle fatigue (HCF) and very high cycle fatigue (VHCF) regimes. Inclusions and pores cause a host of issues that can be broadly grouped into two categories:

(a) Issues related to the inclusion/pore density. These include matters such as large scatter in HCF and VHCF fatigue life data, specimen size dependence [1–3], and surface to bulk transition of HCF and VHCF crack origins. The experimental approaches to the study of these issues require large numbers of experiments to be conducted for large-scale components, and this is not yet practical due to time and cost limitations.

(b) Issues related to the fatigue crack formation processes. These include matters such as the nature of fatigue crack initiation from inclusions and pores, and their effects on the next stages of fatigue crack growth in the matrix. Experimental approaches to the study of these issues are particularly challenging due to the complex and subtle nature of the underlying processes.

The issues in both categories pose questions that are probabilistic in nature and appropriate statistics must be pursued to characterize the underlying variability.

Fatigue life variability naturally exists in all fatigue regimes due to variability in the microstructure and to uncontrolled test conditions. Nevertheless, HCF experiments on advanced metallic alloys, such as Ni-base superalloys, titanium alloys, and high-strength steels, show that fatigue life can be unexpectedly much higher for some specimens [4–18] for which the failure origin is located in the bulk. The relative number of such observations increases as the stress amplitude decreases in the HCF regime and beyond, such that just below the traditional HCF limit, fatigue life data appears to be distributed between two branches. The occurrence of these two distinct failure distributions for surface and bulk initiation sites has been referred to as "Competing Failure Modes" [19,18].

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This duplex distribution manifests as a plateau in the corresponding *S–N* curve, often referred to as a step-wise or duplex *S–N* curve. A bilinear, step-wise, or duplex cumulative distribution function (CDF) of fatigue life is of comparable character; when two separate normal distribution functions are fitted to the datasets, they appear as two linear segments when the CDF of fatigue life is plotted in a logarithmic scale.

As the stress amplitude decreases below the traditional endurance limit and into the VHCF regime, the dataset that corresponds to shorter fatigue lives becomes sparsely populated, whereas the other dataset grows in number and thus controls the mean of the entire data population [12–15,20,21]. As such, the two datasets have been referred to [22] as (1) life-limiting and (2) mean-controlling, respectively. At a given stress amplitude, the overall fatigue life variability can be associated with two sources (c.f. Fig. 1):

1. Variability within each life data population corresponding to surface and internally originated fatigue failures due to variability in the microstructure and underlying mechanisms. This is schematically shown in Fig. 1 by scatter bars (b) and (c) for surface and internally originated fatigue failures, respectively.
2. Separation between the life data populations (as shown schematically in Fig. 1 by gap (a)) due to the:
   i. Environmentally-enhanced cracking at surface sites [18]. Fatigue crack growth rates in air are significantly faster than those observed in vacuum [23,24]. In contrast, the bulk initiation mechanisms essentially operate in a quasi-vacuum environment, associated with longer fatigue lives.
   ii. Lack of constraints on plastic strain localization from adjacent material at sites near the free surface.

Additionally, these life-limiting and mean-controlling datasets broaden and diverge with decreases in the stress amplitude. The increase in overall fatigue life variability due to a decrease in stress amplitude from $\sigma_a = \sigma$ to $\sigma_a = \sigma'$ (within the VHCF regime) can be identified, as schematically shown in Fig. 1.

1. Increased variability ($b < b'$) and ($c < c'$) within lifetime data populations, corresponding to surface and internally originated fatigue failures [12] that arise from increased deformation heterogeneity at lower stress amplitudes.
2. Increased separation ($a < a'$) between the lifetime data populations, attributed to differences in the failure initiation mechanisms to decreases in the stress amplitude [14,18].

Here, the fatigue life is defined by the number of cycles to fracture the specimen, N_f, and normal distribution functions are shown for each data population only for illustrative purposes.

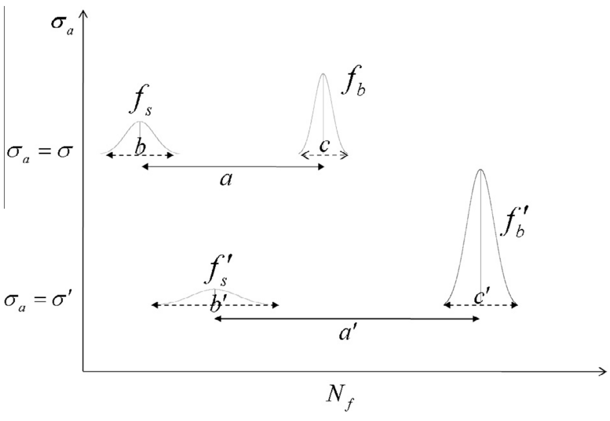

Fig. 1. Variability in fatigue life for a given applied stress/strain amplitude (symbols are for illustrative purposes and do not represent actual experimental data).

It has been shown that the two modes of failure are separable by statistical analysis, even in circumstances where there may be a high degree of overlap in the fatigue data for the two modes [18]. In light of the distinct nature of surface- and bulk-originated failure modes, bimodal representation of the probability distribution function (PDF) of the fatigue life data, $f(x)$, has been suggested as a more accurate fit to the experimental fatigue data [12–14,16,25,26]. In the bimodal representation, two PDFs, f_s and f_b, corresponding to each of the two fatigue life datasets are superimposed as

$$f(x) = p_s f_s(x) + p_b f_b(x) \tag{1}$$

Similarly and in terms of the cumulative distribution function (CDF) of the fatigue life data, $c(x)$,

$$c(x) = p_s c_s(x) + p_b c_b(x) \tag{2}$$

where c_s and c_b correspond to the CDF of each fatigue life dataset. The processes of fatigue crack formation and early growth from surface grains, inclusions, or pores and bulk inclusions govern the scatter within the life-limiting and mean-controlling datasets [27].

Statistically-speaking, the weighting parameters p_s and p_b are the respective probabilities that a given data point belongs to the life-limiting dataset (first term) or the mean-controlling dataset (second term). In clean alloys with inclusions, such as powder metallurgy (PM) processed alloys, the weighting parameters, p_s and p_b of the bimodal fatigue life distribution in Eqs. (1) and (2) can be more specifically referred to as the probability of failure initiation (formation and early growth) from surface and bulk inclusions, respectively.

Physically-based life prediction methodologies that integrate the mechanisms of fatigue variability in Ni-base superalloys are of great interest in life-extension as well as in new alloy development in the gas turbine industry [27–29]. To that end, life-limiting mechanisms of superalloys have received considerable attention [12,14,20,23,24,27,30,31]. This is because superalloy applications such as in aircraft gas turbine applications demand a very low probability of failure.

Many experimental studies seek to enhance the general understanding of fatigue crack formation and early growth behavior by obtaining extensive databases of fatigue crack growth data, often introducing known populations of artificial inclusions (seeds) to production powder to intentionally promote surface fatigue failure initiation. Such experiments are very time-consuming, expensive, and difficult. Although a fair level of understanding has been achieved regarding small crack growth behavior, a physically-based predictive tool is lacking, and fatigue crack growth behavior is still the subject of active research.

Besides the distribution of the life-limiting dataset, the populations of life-limiting versus mean-controlling distributions (i.e., surface versus bulk initiation probabilities in the PM-processed Ni-Base superalloys), p_s/p_b, can significantly impact the extrapolated low failure probability estimate of fatigue life for the bimodal fatigue life distribution. To illustrate this effect, we cite the experimental fatigue data of a study on subsolvus PM-processed IN100 tested at 650 °C, $f = 0.33$ Hz, and $R_\sigma = 0.05$ (stress-controlled) [14]. The cumulative distribution function (CDF) of fatigue life data [14] is shown in blue symbols in Fig. 2. The fitted CDF to all the fatigue life data points is shown with a continuous blue line. The traditional B0.1 estimate (1 in 1000 probability of failure) is computed by extrapolating this fitted CDF. The CDF of the simulated life-limiting distribution, i.e., c_s in Eq. (2) is shown with a black dashed line in Fig. 2. In the referenced study, the life-limiting distribution was simulated using the Paris crack growth equation with randomized parameters and an equivalent initial crack size for the inclusion initiated cracks. By simulating the distribution c_s, a first-order improvement to the B0.1 estimate is achieved by