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A plasticity-corrected stress intensity factor for fatigue crack growth in ductile materials under cyclic compression

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ABSTRACT

For prediction of the fatigue crack growth (FCG) behavior under cyclic compression, a plasticity-corrected stress intensity factor (PC-SIF) range ΔK_{pc} is proposed on the basis of plastic zone toughening theory. The FCG behaviors in cyclic compression, and the effects of load ratio, preloading and mean load, are well predicted by this new mechanical driving force parameter. Comparisons with experimental data showed that the proposed PC-SIF range ΔK_{pc} is an effective single mechanical parameter capable of describing the FCG behavior under different cyclic compressive loading conditions.

1. Introduction

It has been extensively reported that under cyclic compressive loading fatigue cracks in ductile materials can nucleate at notch root of engineering components and grow at a diminishing rate until it arrests [1–10]. Plastic deformation of the notch root has been recognized as a dominant mechanism for the fatigue crack initiation and growth [1–5]. This can be illustrated in Fig. 1 which shows schematically the stress distribution in front of a notch for an elastic perfectly-plastic material both under cyclic compression and cyclic tension. Because of plastic deformation at the notch root, a residual tensile stress would be introduced upon unloading in both cyclic tension and cyclic compression. A cyclic plastic zone would be formed during forward and reversal loadings. It is worth noting that under cyclic compression the stress intensity factor (SIF) concept at the notch tip has the physical meaning as in cyclic tension as long as the notch surfaces do not contact each other [2]. Evidently, if the elastic SIF K_{el}^{max} in tension is equal to $-K_{el}^{min}$ in compression and $\Delta K_{el}(= K_{el}^{max} - K_{el}^{min}$ for both cases) is equal, the stresses at $-K_{el}^{min}$ in compression have the opposite signs as at K_{el}^{max} in tension, and also for the case of K_{el}^{min} in tension and K_{el}^{max} in compression. Thus, the size of the cyclic plastic zone is equal in cyclic compression and cyclic tension. Detailed finite element (FE) analysis [3] showed that the stress in the cyclic plastic zone undergoes approximately symmetrical tension–compression cycles during cyclic compression. Thus, under cyclic compression the tensile stress in the cyclic plastic zone is responsible for crack initiation and growth at the notch tip.

It is well known that the elastic SIF range ΔK_{el} has been widely taken as a crack driving force for prediction of the fatigue crack growth (FCG) rate by Paris Law. However, according to ASTM-E647 the compression part of the fatigue loading is ignored in the calculation of ΔK_{el} (i.e., $\Delta K_{el} = K_{el}^{max}$ in tension–compression loading). This stems from the assumption that the crack surfaces are completely in contact or closed during compression part of the fatigue loading. However, it has been observed that the compression part of a fatigue loading has a significant effect on the FCG behavior [11]. How to account for this effect is one of the most puzzling questions in fatigue. Therefore, up to now most of the previous analyses on pure cyclic compression were limited on determining the tensile stress in the cyclic plastic zone in order to establish a threshold stress range responsible for crack initiation and growth at the notch root [3,5–8], rather than on the SIF concept as used in cyclic tension.

In the present study, we introduce a plasticity-corrected (PC) SIF range, ΔK_{pc}, as a crack driven force for prediction of the FCG behavior under cyclic compression loading.

2. The PC-SIF range ΔK_{pc}

As aforementioned, crack initiation and growth under cyclic compression is a plasticity-induced phenomenon. When plastic zone occurs around the crack tip, Paris law suffers from two major drawbacks in its application to ductile fracture: the lack of explicit dependence on the maximum load or load ratio (that control the

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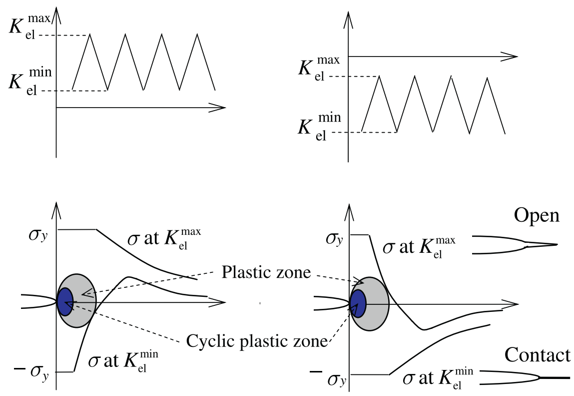

(a) Cyclic tension (b) Cyclic compression

Fig. 1. Schematic stress distribution in front of sharp notch for an elastic-perfectly plastic material: (a) cyclic tension and (b) cyclic compression.

extent of plasticity within each cycle), and the lack of dependence on the loading history [12]. To date there is not a well-recognized method of correlating Paris law with crack-tip plastic deformation. The key reason is that a sound mechanical model has not yet been established for quantitative evaluation of the effects of the crack-tip plastic zone on the SIF.

Recently, Li and his co-authors [13–16] demonstrated that a plastically deformed zone around a stressed crack tip can be identified with a homogenous inclusion of transformation strain by means of Eshelby equivalent inclusion method. Thus, the crack-plastic zone interaction can then be determined based on Hutchinson's general solution for crack-transformation strain interaction [17]. Hutchinson's solution provides a theoretical basis of the stress-induced transformation toughening in elastic solids [18,19] and the plastic zone toughening in ductile materials [13–16].

According to this plastic zone toughening theory, closed-form solutions for change in SIF due to crack-tip plastic zone are given both for mode-I and mode-II cracks under small scale yielding conditions. For mode I crack the solution is expressed as

$$\Delta K_{pl} = \frac{1}{2\sqrt{2\pi}}\int_\Omega r^{-3/2}\left[\frac{K_{el}}{\sqrt{2\pi r}}\left(2\cos\frac{\theta}{2}\cos\frac{3\theta}{2}+3\sin^2\theta\cos\theta\right)\right. \\ +3(\sigma_{11}-\sigma_{22})\sin\theta\sin\frac{5\theta}{2}-6\sigma_{12}\sin\theta\cos\frac{5\theta}{2} \\ \left.-(\sigma_{11}+\sigma_{22})\cos\frac{3\theta}{2}\right]d\Omega, \tag{1}$$

for plane strain condition [15,20]. In Eq. (1) K_{el} is the remotely applied elastic ISF, Ω is the area of the plastic zone around the crack tip, σ_{11}, σ_{22}, σ_{12} are the stresses in the plastic zone. Eq. (1) indicates that if the distributions of the stresses in the plastic zone, the shape and size of the plastic zone are known, the change in the SIF due to the plastic deformation can be evaluated. Accordingly, a PC-SIF, K_{pc}, can be defined as

$$K_{pc} = K_{el} + \Delta K_{pl}. \tag{2}$$

For a fatigue crack, the PC-ISF range ΔK_{pc} can be expressed as

$$\Delta K_{pc} = K_{pc}^{max} - K_{pc}^{min}, \tag{3}$$

where K_{pc}^{max} and K_{pc}^{min} are the PC-SIF associated with the maximum and minimum load in a loading cycle according to Eq. (2). Crack initiation and growth under cyclic compression is completely a plasticity-induced phenomenon. It is therefore expected that it can be actually described by the new crack driving force parameter.

3. The calculation of the ΔK_{pc} under cyclic compression

As shown in Fig. 1, there is no substantial difference for the stress distributions in front of a sharp notch under cyclic tension and cyclic compression. However, in cyclic compression the crack propagates in such a way that the cracked surfaces completely contact at K_{el}^{min}, while completely open at K_{el}^{max}, as shown in Fig. 1b. The former may be quite different from the cyclic tension. The concept of the SIF at an open crack tip has a definite meaning both for compression and tension, but loss its significance at a closed crack tip.

Therefore, under cyclic compression the effective crack tip at K_{el}^{min} is always located at the notch tip of crack length $a \equiv a_0$, while that at K_{el}^{max} is located at $a = a_0 + \Delta a$. Thus, at the maximum load the PC-SIF is given by

$$K_{pc}^{max} = K_{el}^{max}(a) + \Delta K_{pl}^{max}(a), \tag{4}$$

in which

$$\Delta K_{pl}^{max}(a) = \frac{1}{2\sqrt{2\pi}}\int_\Omega r^{-3/2}\left[\frac{K_{el}^{max}(a)}{\sqrt{2\pi r}}\left(2\cos\frac{\theta}{2}\cos\frac{3\theta}{2}+3\sin^2\theta\cos\theta\right)\right. \\ +3(\sigma_{11}-\sigma_{22})\sin\theta\sin\frac{5\theta}{2}-6\sigma_{12}\sin\theta\cos\frac{5\theta}{2} \\ \left.-(\sigma_{11}+\sigma_{22})\cos\frac{3\theta}{2}\right]d\Omega, \tag{5}$$

while that at the minimum load is expressed as

$$K_{pc}^{min} = K_{el}^{min}(a_0) + \Delta K_{pl}^{min}(a_0), \tag{6}$$

where

$$\Delta K_{pl}^{min}(a_0) = \frac{1}{2\sqrt{2\pi}}\int_\Omega r^{-3/2}\left[\frac{K_{el}^{min}(a_0)}{\sqrt{2\pi r}}\left(2\cos\frac{\theta}{2}\cos\frac{3\theta}{2}+3\sin^2\theta\cos\theta\right)\right. \\ +3(\sigma_{11}-\sigma_{22})\sin\theta\sin\frac{5\theta}{2}-6\sigma_{12}\sin\theta\cos\frac{5\theta}{2} \\ \left.-(\sigma_{11}+\sigma_{22})\cos\frac{3\theta}{2}\right]d\Omega. \tag{7}$$

Then we have

$$\Delta K_{pc} = K_{pc}^{max}(a) - K_{pc}^{min}(a_0). \tag{8}$$

Note that the $K_{el}^{min}(a_0)$ in Eq. (7) is not changed at constant amplitude compression. However, during crack growth the stress in the plastic zone changes its value, thereby change in the value of $\Delta K_{pl}^{min}(a_0)$.

It is important to note that under cyclic loading the plastic deformation around crack tip retains a "memory" of prior deformation that affects its subsequent responses. The non-uniformly distributed permanent inelastic strains within the plastic zone lead to the creation of residual stresses, which modify the crack-tip stress field and therefore change the K_{pc}. Thus, during cyclic loading the PC-SIF-range ΔK_{pc} depends on the loading history. The memory effect of the plastic deformation requires that once a material element around the notch tip is plastically deformed, it will be taken as a part of the current plastic zone even if this material element deforms elastically during the following loading. Thus, for a growing crack the area of the plastic zone, the Ω in Eqs. (5) and (7), includes the plastically deformed wake zone.

In addition, FE-simulation on FCG [21] showed that very strong mesh distortion in front of the crack tip occurs, especially for the mesh with small size, leading to unacceptable values of stress field. On the other hand, in the region closer to the crack tip there is a fatigue damage zone. In general, what is happening in such a damage zone is considered as a shearing decohesion or void formation caused by severe local shearing processes in ductile materials. To avoid this numerical difficulty and meet the actual damage phenomenon, a small circular region around crack tip of radius