International Journal of Fatigue 33 (2011) 255–264

Contents lists available at ScienceDirect

International Journal of Fatigue

journal homepage: www.elsevier.com/locate/ijfatigue

Fatigue damage accumulation modelling in the range of complex low-cycle loadings – The strain approach and its experimental verification on the basis of EN AW-2007 aluminum alloy

J. Szusta *, A. Seweryn

Faculty of Mechanical Engineering, Bialystok University of Technology, 45C Wiejska Str., 15-351 Bialystok, Poland

ARTICLE INFO

Article history:
Received 29 October 2009
Received in revised form 17 August 2010
Accepted 25 August 2010
Available online 31 August 2010

Keywords:
Low-cycle fatigue life
Damage accumulation
Material hardening
Crack initiation
Complex loading
Experimental tests

ABSTRACT

The paper presents the fatigue damage accumulation model created to analyse fatigue life of structure elements operating in conditions of multiaxial, non-proportional low-cycle loadings. The authors used the approach connected with the critical plane in the presented model. In the discussed approach the damage variable depends on the stress damage accumulation function and the increments of plastic non-dilatational strain. The components of the stress and strain tensor in the conditions of complex low-cycle loadings were determined using constitutive relations and the law of kinematic hardening. It was assumed that cracks occur when either the normal stress or damage variable on any physical plane reaches critical values. The presented model was verified experimentally on the base of EN AW-2007 aluminum alloy. The main body of tests were performed in the conditions of biaxial loadings (tension–compression and torsion), proportional and non-proportional, according to the assumed research program, on research stands which allow performing complex loading histories. These were preceded by uniaxial loading state tests (cyclic tension–compression or torsion) on the basis of which parameters of the calculation model should be calculated.

1. Introduction

In the classical approach to damage accumulation modelling in a material a few subsequently occurring stages can be distinguished:

Stage 0 – preparing calculation data (determining the shape, boundary and initial conditions, loading histories of a structure element – on the basis of exploitation conditions).
Stage I – the analysis of stress and strain fields and their changeability (the use of constitutive relations and numerical methods, including: finite element method, (FEM), Boundary element method (BEM) or finite differences method (FDM)).
Stage II – damage accumulation modelling in a material (determining the weakest spot in a structure element, calculating the critical loading parameters, including: the value, operating time or the number of cycles to macro-crack initiation).
Stage III – predicting cracks in a structure element (calculation of crack trajectory and the rate of crack development, determining the critical conditions of total damage of a structure element).

Fatigue damage accumulation modelling is more complicated in the case of a small number of loading cycles when there occurs permanent strain. Fatigue life tests of this type have been conducted for only half a century. The forerunners of this type of research were Coffin [4] and Manson [5] who proposed the first calculation dependencies in 1954. At present, this type of research is conducted on a large scale at numerous research centres all over the world [1–3,6,7]. This happens for economical reasons and high standards required in the field of engineering structures.

Fatigue criteria in the case of multiaxial loadings often focus on determining such equivalent value which makes it possible to compare this type of loading with uniaxial loading. It is then assumed that fatigue damage accumulation will be the same for uniaxial and multiaxial loading if the equivalent values for the two types of loading are identical. Typically, maximum non-dilatational strain [8–10], maximum shearing strain [11] or the energy dissipated in a material [12,13] are assumed as comparative values.

The best known criterion is that of Manson–Coffin (later modified by Morrow [14]), which has a wide practical application in the analysis of damage accumulation in the conditions of low-cycle fatigue loadings. This criterion forms the base for formulation of damage accumulation models in complex loading states. When it is described with the use of the equivalent strain $\Delta\varepsilon_{eq}$ (in the sense of Huber–von Mises) it has the form (Fig. 1):

* Corresponding author. Tel.: +48 085 746 9300; fax: +48 085 7469210.
E-mail addresses: j.szusta@pb.edu.pl, szusta@pb.bialystok.pl (J. Szusta), a.seweryn@pb.edu.pl (A. Seweryn).

0142-1123/$ - see front matter © 2010 Elsevier Ltd. All rights reserved.
doi:10.1016/j.ijfatigue.2010.08.013

$$\frac{\Delta\varepsilon_{eq}}{2} = \frac{\sigma_f}{E}(2N_f)^b + \varepsilon_f(2N_f)^c, \quad (1)$$

where $\varepsilon_{eq} = \left(\frac{2}{3}e_{ij}e_{ij}\right)^{1/2}$, e_{ij} – the components of strain deviator, N_f – the number of cycles to failure, E – Young's module, σ_f, b – coefficient (critical stress during tension) and the exponent of fatigue damage curve respectively, ε_f, c – coefficient (critical strain) and exponent of plastic strain fatigue curve, respectively.

Despite a large number of damage accumulation criteria in complex loading states at a small number of cycles, none of those criteria became widely accepted. Applying these criteria requires great caution and ought to be limited to the experimentally verified special cases. Thus, there is a justified need to create a damage accumulation model with which it would be possible to predict fatigue life in a larger spectrum of loading cases in the range of a small number of cycles.

2. Material hardening model

The model described in the present paper consists of two essential parts.

Part one contains constitutive relations:

- generalized Hook's law;
- gradient flow rule associated with plasticity condition of Huber–von-Mises;
- kinematic material hardening law.

Part two includes:
- the law of damage accumulation;
- the criterion of fatigue crack initiation.

In order to determine fatigue life of a structure element one must have knowledge of stress and strain fields and their changeability. These are determined by applying constitutive relations and numerical methods, especially in the case of non-homogeneous stress and strain fields. In the presented calculation model the authors used physical relations (generalized Hook's law and gradient flow rule combined with plasticity condition) in incremental form, namely:

$$\begin{aligned} d\varepsilon_{ij} &= d\varepsilon^e_{ij} + d\varepsilon^p_{ij}, \\ d\varepsilon^e_{ij} &= \tfrac{1+\nu}{E}d\sigma_{ij} - \tfrac{\nu}{E}d\sigma_{kk}\delta_{ij}, \\ d\varepsilon^p_{ij} &= d\lambda\tfrac{\partial f}{\partial\sigma_{ij}}. \end{aligned} \quad (2)$$

where $d\varepsilon_{ij}$, $d\sigma_{ij}$ – the increments of the stress and strain tensor components respectively, $d\varepsilon^e_{ij}$, $d\varepsilon^p_{ij}$ – the increments of the elastic and plastic strain tensor components respectively, $d\lambda$ – proportionality coefficient, E – Young's module, ν – Poisson ratio, δ_{ij} – Kronecker's delta

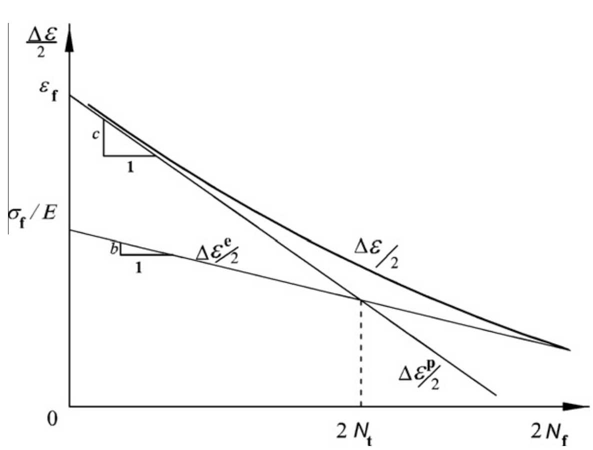

Fig. 1. Manson–Coffin curve; the axes in the logarithmic scale.

Plastic surface $f = 0$ was determined with Huber–von Mises condition:

$$f = \frac{3}{2}(s_{ij} - \alpha_{ij})(s_{ij} - \alpha_{ij}) - R^2 = 0, \quad (3)$$

where s_{ij} – the components of stress deviator; α_{ij} – components of the tensor which determines plasticity surface translation; R – the size of plastic surface.

The effect of material hardening was described using Mroz's multisurface model [15] with Garud's modifications [16].

The Mroz with Garud's modification multisurface model of the material hardening gives the best results in the case of nonproportional loadings. The above mentioned formulation of the kinematic hardening of the material makes possible the imitation of the material anisotropy coming into being in the figure of the Bauschinger effect. In the cases of uniaxial and proportional loading, the described law makes possible the elimination of Masing rule (the leaning on the transformation scale of reference system during the description of the cyclic deformation) during the analysis of the material unloading reverse.

The detailed description of determining the components of stress and strain tensors in the presented case can be found in, for example, a paper by Seweryn et al. [7].

3. Fatigue damage accumulation model

Most researchers assume that the permanent plait slips (Garud [16]) are the spots where defects originate when structure elements are subjected to low-cycle loadings, while crack initiation is the result of accumulation of those defects. Slip lines and plait slips which consist of those lines are visible evidence of plastic strain. They initially appear in the grains which are most conveniently orientated in relation to the active stress system. This conclusion was made on the basis of numerous tests, in the course of which it was observed that as a result of cyclic stress changes, even by values lower than the permanent fatigue life. The tests were performed on the polished surfaces of the samples (made from mono- and polycrystalline metals) where slip lines first appear which then transform into fatigue plait slips (numerous parallel gaps) along with the increase of the number of loading cycles Han et al. [17].

Presented in the paper [7] the damage accumulation model described the change of the damaged material properties with using isotropic damage accumulation measure connected with increase of energy dissipated in the material. Such approach is sufficient in the case of uniaxial loading. However complex loading force the anisotropic distribution of damages and require using the damage accumulation measure connected with the physical plane. Such approach presented in this paper allows to define the direction of the crack initiation. In the analysed case anisotropic damage accumulation measure connected with the increase of plastic non-dilatational strain on the physical plane. This solution causes extension of the analysis time because it considers all physical planes on which the damage accumulation takes place. It accumulates independently these damages for every physical planes, after that checks on which physical planes the crack initiation condition was reached critical value the most quickly.

In the proposed model it was assumed that slips in grains, and indirectly non-dilatational plastic strain, are responsible for damage accumulation as well as incubation and development of microcracks in the conditions of low-cycle loadings in polycrystalline materials. Due to that the law of damage accumulation was formulated incrementally. The increment of damage variable $d\omega_n$ caused by the development of plastic strain was related to the stress