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Short Communication

A note on Hunt and Crossley model with generalized visco-elastic damping

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ABSTRACT

The Hunt and Crossley model has been demonstrated to give accurate predictions of contact force generated by the impact of a solid object. The simulated contact force values have been demonstrated to match closely with experimental results as reported in the literature. However, derivation of the expression for finding the value of the damping coefficient has never been presented. The writing of this paper was motivated by the need to fill in the important knowledge gap.

1. Introduction

This paper is relevant to analysis of impact actions by falling or moving objects in the context of sports [1–3], vehicular accidents [4–7], windborne debris [8] or hail [9]. The focus of interest is in the estimation of the contact force (F_c) which is the controlling parameter for localised actions of the impact causing denting, crushing or perforation of the surface of the target [10].

Accurate simulation of the contact force relies on the choice of the adopted contact law as well as the associated input parameters. Hunt and Crossley model which is non-linear visco-elastically behaved as depicted in Fig. 1 and represented by Eq. (1), is based on the use of non-linear damper in conjunction with the Hertzian spring [11–13]. The dynamic compressive stiffness K, non-linear power exponent n, and damping coefficient D are parameters characterizing the conditions at contact. The Hunt & Crossley model features the use of three coefficients: K, n and D to fully define the transient force at point of contact between the impactor object and the surface of the target as function of the relative displacement and velocity of the two objects. These parameter values need to be calibrated against experimental measurements in order that real behaviour at contact is represented accurately. However, uncertainties over the value of these coefficients have always been an issue given the difficulties to have values of all three coefficients calibrated to achieve the best match with experimental measurements. The issues have been circumvented by the use of a closed form expression which provides prediction of the value of D for any given values of K, n and coefficient of restitution C_e (the 3rd item on the list can be inferred from measurements taken from an impact experiment). Given this new closed form expression only two coefficients (K and n) need be obtained by calibration. This new development is instrumental in having design charts drawn for determining the value of the contact force for the whole range of scenarios of storm debris [8] and hail [9] but no presentation of the basis of the adopted closed form relationship can be found in the literature. This paper presents for the first time the formal derivation for this important closed form expression along with an illustration of its use. Contact force associated with large indentation and high energy dissipation can be predicted by the adopted model [14,15].

$$F_c = K\delta^n + D\delta^n\dot{\delta} \tag{1}$$

where $\delta\,(=x_i - x_j)$ is the indentation of the impactor into the surface of the target (in which x_i is the movements of the impactor and x_j is the displacements of the target); $\dot{\delta}\,(=v_i - v_j)$ is the indentation velocity (in which v_i and v_j are velocities of impactor and target at any time in the course of the impact).

Solution for the damping coefficient D forming part of the Hunt and Crossley model has been interpreted in many different ways for characterizing energy losses that can occur in the course of an impact [14,16–21]. The value of D depends on both the material properties of the impactor object and the intensity of impact. Eq. (2), which is based on the simplified assumption of linear visco-elastic behaviour at contact (where n is equal to 1.0), is applicable to scenarios where the area of contact is relatively small compared with the dimensions of the impactor object [22–25]. Flores et al. [26] proposed an alternative formula for determining the value of D as represented by Eq. (3) which fixed the value of n at 1.5 as per Hertz law [27–37].

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Nomenclature

i, j	impactor, target
m_i, m_j, m	impactor mass, target mass, equivalent mass
x_i, x_j	displacement of impactor or target
v_i, v_j, V	velocity of impactor or target, common velocity
$v_i^{(-)}, v_j^{(-)}$	initial impact velocity of impactor or target
$v_i^{(+)}, v_j^{(+)}$	separation velocity of impactor or target
$\delta, \delta^{(-)}, \delta_m$	indentation, initial indentation, maximum indentation
$\dot{\delta}, \dot{\delta}^{(-)}, \dot{\delta}^{(+)}$	indentation velocity, initial indentation velocity, separation indentation velocity
$\ddot{\delta}$	indentation acceleration
$t^{(-)}, t^{(m)}, t^{(+)}$	time of initial contact, time of maximum indentation, time of separation
C_e	coefficient of restitution
F_c	contact force
n	non-linear power exponent
K	dynamic compressive stiffness
k	equivalent stiffness
D	damping coefficient
c	equivalent damping coefficient
ζ	damping ratio
ω	radial frequency
ω_d	damped radial frequency
ΔE_{loss}	total amount of energy loss occurring on impact

$$D = \frac{3}{2}\left(\frac{1-C_e}{C_e}\right)\frac{K}{\dot{\delta}^{(-)}} \tag{2}$$

$$D = \frac{8}{5}\left(\frac{1-C_e}{C_e}\right)\frac{K}{\dot{\delta}^{(-)}} \tag{3}$$

$$\text{where } C_e = -\frac{\dot{\delta}^{(+)}}{\dot{\delta}^{(-)}} \tag{4}$$

where $\dot{\delta}^{(-)}(=v_i^{(-)} - v_j^{(-)})$ and $\dot{\delta}^{(+)}(=v_i^{(+)} - v_j^{(+)})$ are indentation velocities immediately prior or following the impact; $v_i^{(-)}$ and $v_j^{(-)}$ are the initial velocities of the impactor and the target immediately prior to the occurrence of impact; $v_i^{(+)}$ and $v_j^{(+)}$ are the respective velocities immediately following the impact.

In summary, the literature has presented solutions for D for the special cases of $n = 1.0$ and 1.5. However, the coefficient n is not fixed at a constant value. Instead, it may take any value within the range: 1–2 depending on the condition of the impact and the compressive properties of the impacting objects [8,38,39]. The value to adopt can be determined by calibration to achieve good match of the modelled contact force with experimental measurements. Turning n from a constant into a variable will result in more realistic prediction of energy dissipation in an impact scenario. The generalized solution for D for any given value of n as described in Eq. (5) has been adopted for simulating contact force as per the Hunt and Crossley model across many impact scenarios involving a range of spherical impactor objects [10], wind-borne debris specimens [8], and hail specimens [9,38,40]. However, the theoretical basis of the introduced relationship has never been explained in the literature. Details of the formal derivation for the expression are presented herein in Section 2 through Section 5 of this article. The writing of this paper was motivated by the need to present the theoretical derivation in a comprehensive manner in order to provide confidence in its usages.

$$D = (0.2n + 1.3) \times \left(\frac{1-C_e}{C_e}\right)\frac{K}{\dot{\delta}^{(-)}} \tag{5}$$

Essentially, the purpose of the closed-form expression of Eq. (5) is to determine the value of the coefficient D once the value of the other two coefficients: K and n are known. The construction of the contact force model as depicted in Fig. 1 based on the governing equation of the Hunt and Crossley model of Eq. (1) is hence expedited. Eq. (5) is also consistent with the previously published expressions (of Eqs. (2) and (3)) that have been derived for the special cases where $n = 1.0$ and $n = 1.5$ for the generalized solution for D.

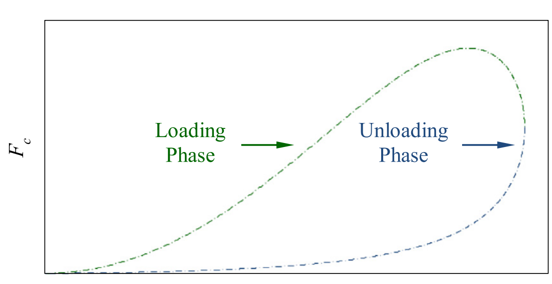

Fig. 1. Hunt and Crossley contact model.

2. Derivation strategy

In the Hunt and Crossley model (refer Eq. (1)), assume all energy losses (ΔE_{loss}) occurring on the impact is taken by the viscous damper ($\oint D\delta^n\dot{\delta}d\delta$). The relationship can be build up as in Eq. (6).

$$\Delta E_{\text{loss}} = \oint D\delta^n\dot{\delta}d\delta \tag{6}$$

Deriving the generalized solution for D involves a 3 step procedure as depicted in Fig. 2 and illustrated in Sections 3–5.

3. Total amount of energy loss during the impact process (ΔE_{loss})

There are two phases in the course of an impact action as presented in Fig. 3: the compression phase and restitution phase. The compression phase commences when two objects (with masses m_i and m_j) come into contact at time $t^{(-)}$, and terminates when the impacting objects travel with common velocity (V) reaching maximum indentation (δ_m) at time $t^{(m)}$; the restitution phase commences as soon as the compression phase comes to an end, and terminates once the two colliding objects separate from each other at time $t^{(+)}$. The loss of energy for the entire impact process has to include the amount of energy dissipated for both the compression phase and the restitution phase.

Considering the condition of equal energy and equal momentum at the onset of the compression phase ($t^{(-)}$) and the termination of the restitution phase ($t^{(+)}$), the following expressions are obtained:

$$\left[\frac{1}{2}m_i(v_i^{(-)})^2 + \frac{1}{2}m_j(v_j^{(-)})^2\right] = \Delta E_{loss} + \left[\frac{1}{2}m_i(v_i^{(+)})^2 + \frac{1}{2}m_j(v_j^{(+)})^2\right] \tag{7}$$

$$m_iv_i^{(-)} + m_jv_j^{(-)} = m_iv_i^{(+)} + m_jv_j^{(+)} \tag{8}$$

By letting $m = \frac{m_im_j}{m_i+m_j}$, Eq. (8) can be presented as follows:

$$m_i = -m\frac{(v_i^{(-)} - v_j^{(-)}) - (v_i^{(+)} - v_j^{(+)})}{(v_i^{(+)} - v_i^{(-)})} \text{ and } m_j = m\frac{(v_i^{(-)} - v_j^{(-)}) - (v_i^{(+)} - v_j^{(+)})}{(v_j^{(+)} - v_j^{(-)})} \tag{9}$$

Submitting Eq. (9) into Eq. (7) results in the following expression:

$$\Delta E_{loss} = \frac{1}{2}m[(v_i^{(-)} - v_j^{(-)})^2 - (v_i^{(+)} - v_j^{(+)})^2] \tag{10}$$

The total amount of energy loss occurring on impact as represented by Eq. (10) can be simplified further into Eq. (11) by considering the