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Impact load wave transmission in elastic metamaterials

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ABSTRACT

Structures that consist of resonating endo-structures within a load bearing exo-structure possess the ability to mitigate dynamic load within a fixed frequency range. Their dynamic behavior can be characterized by the enactment of negative effective mass density. This research work presents the design, fabrication and dynamic load characterization of a locally resonant elastic metamaterial. Mass–spring system with negative effective mass is experimentally investigated and numerically analyzed, with the aim to reveal stress wave transmission property in low-frequency range under impact loading. Local resonance frequencies of the basic unit cells are designed and generated using springs with different spring constants. Results evidently show that impact load mitigation occurs in the presence of internal resonators. Parametric studies reveal that wave attenuation performance is improved by using a variety of resonance frequencies in the local resonators design. In addition, alternating the arrangement of different local resonators have negligible influence on wave attenuation. A good agreement between numerical simulation and experimental testing is achieved. The significance of this work lies in the experimental and numerical evaluation of wave attenuation efficiency in elastic metamaterials, thus providing a strong foundation and design basis in the future potential of elastic metamaterials for impact protection applications.

1. Introduction

It is widely known that dynamic loadings like impact loads on engineering structures could result in damaging effects ranging from degraded material properties of the original structure to catastrophic failure of the entire structure. Many solutions have been utilized to reduce these ill effects including layered or periodic structures [1,2], sacrificial structures of foams or cellular solids [3], energy dissipation by bi-stable elements [4], damping layer treatment of structures [5], adjusted vibration absorbers [6–8], nonlinear energy sinks [9] and negative mass structure (NMS) [10]. Negative effective mass structures are constructed by placing resonating internal mass within a load bearable external mass to mitigate impact wave. These structures are motivated by stress wave reduction capabilities of locally resonant metamaterials which exhibit negative effective mass density over a particular frequency range [11].

Metamaterials are envisioned to play a vital role in mitigating of stress waves under dynamic loads. They are fabricated from specially engineered assemblies of multiple elements. These elements are designed using common materials such as metals or plastics. A wide range of metamaterials can be found including acoustic elastic (AE) metamaterials like locally resonant phononic crystals [12], periodic Helmholtz resonant cavities [13]; and electromagnetic (EM) metamaterials like periodic wires and split ring resonators [14]. Some potential applications of AE metamaterials are in sound filtering, cloaking device [15] and seismic protection [16]. Metamaterials have unconventional properties which are not very common in nature. Properties and controlling parameters of metamaterials can be tailored to specific values by tuning their microstructures. For example, locally resonant phononic crystals exhibit negative mass density at certain frequencies when their sub-wavelength microstructures resonate and move out of phase with the external stimulation [12].

Under external excitation, most materials found in nature usually react in phase with the excitation. However, by carefully designing their microstructures, elastic metamaterials can be fabricated to react out of phase due to resonant mechanism. This results in negative effective mass density behavior. Negative mass density can be realized through a simple mass-spring system described by Milton and Willis [17]. They proposed that the dynamic effective mass of composite material should be defined by Newton's law of motion, as opposed to the static gravitational mass. Their studies revealed that composite materials with tailored microstructures can show a negative momentum for a positive momentum stimulation. This indicates performance of a negative effective mass structure.

The novel dynamic characteristics of structural systems with resonators has been familiar for more than a century. One of the earliest

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work of the band gap effect for acoustic waves was shown by Vincent [18] in 1898 using nonlinear oscillators. Still, there has been a resurging interest in recent years [19–23] in wave attenuation using local resonators of heavy core and light external mass connected by springs, dashpots, rubber, cantilever beam, soft coatings etc. Impact wave mitigation occurs through the presence of band gaps, where the input frequency matches with the local resonance band gap frequency range of the internal structure (internal mass connected with spring). At this moment, significant amount of incident energy is transferred to internal resonators temporally. Milton and Willis [17] examined the frequency dependent behavior of the mass density of materials and found that the effective mass could be negative with resonating inclusions. Huang et al. [19] presented different analytical models to examine the negative effective mass phenomenon in mass-in-mass lattice systems. Experimental validation of the negative effective mass and low transmission of elastic waves in the band gap frequency range was shown by Yao et al. [20].

Recently, Baravelli et al. [21] analyzed the suppression of the first resonance mode in backbone frame. They used an acoustic metamaterial inclusion made of viscoelastic resonant lattices with chiral topology and cylindrical mass insertions. It was shown that if properly tuned, internal resonators can show out-of-phase motion of the inclusion with respect to the primary structure in a desired frequency range. This was the reason for the suppression of vibrational energy. Wave mitigation and energy transfer mechanism in the band gap for locally resonant mass-in-mass AE metamaterial was studied by Huang and Sun [22]. They concluded that as soon as impinging frequency equals the local resonance frequency (LRF) of the internal resonator, most of the energy of external force on the system is stored by the internal mass as kinetic energy. However, the energy stored by the internal mass is temporary and is later taken out by the external forcing agency in the form of negative work in a cyclic manner. This underlying mechanism of sequestration and rejection of energy achieved using resonators confirms an apparent damping ability in such elastic structures. This unconventional property can surpass those achieved from dissipative mechanisms in structures with conventional damping. Moreover, broadband attenuation is achievable using locally dissipative AE metamaterials by adjusting the dissipative and resonance mechanisms of the locally damped oscillator attachments. As a result, metamaterial-inspired negative effective mass structure (NMS) offers the potential of achieving much higher simultaneous stiffness and damping-like characteristics than conventionally damped structures [10].

Tan et al. [11] presented the effects of resonators configuration on the impact wave attenuation based on numerical studies. However, there is not yet any experimental validation of the work. Sharma and Sun [23] related the bending strain reduction to impact load duration and resonator frequencies. However, their experimental work did not show direct relation between impact wave transmission performances to impact energy levels.

In this work, we will experimentally examine the impact wave attenuation using locally resonant elastic metamaterial spring-mass system suggested by Milton and Willis [17] in the low frequency region, in contrast to high frequency or ultrasonic domains. The main reason being that low frequency regime is more relevant and applicable to impact damage phenomenon which generally occurs at low frequency region (100–2000 Hz). In addition, we will develop a numerical model and perform parametric studies to observe the influence of local resonator designs on the impact wave attenuation performance of elastic metamaterials.

2. Analytical mass-in-mass model and negative effective mass

This section briefly explains the theory of negative effective mass density obtained by using a mass-in-mass spring system [17,19]. Such a system consists of an outer mass, m_1, and an inner mass, m_2, connected by a spring of stiffness k_2, as shown in Fig. 1.

Here, the external applied force F and masses displacements u_γ ($\gamma = 1, 2$) are influenced by harmonic motion, as shown in Eqs. (1) and (2). Using the harmonic wave behavior, we derive equations in terms of the applied force.

$$F(t) = F_0 e^{-i\omega t} \quad (1)$$

$$u_\gamma(t) = \hat{u}_\gamma e^{-i\omega t} \quad \gamma = 1, 2 \quad (2)$$

From Newton's second law and free body diagram for each of the masses, we obtain

$$m_1 a_1 = F + k_2(u_2 - u_1) \quad (3)$$

where acceleration $a_1 = \ddot{u}_1$, m_1 is the outer mass.

$$m_2 a_2 = k_2(u_1 - u_2) \quad (4)$$

where acceleration, $a_2 = \ddot{u}_2$, m_2 is the inner mass.

Solving the above equations using the harmonic wave behavior, we simplify to

$$F_0 = -\left(m_1 + \frac{\omega_2^2 m_2}{\omega_2^2 - \omega^2}\right)\omega^2 \hat{u}_1 \quad (5)$$

where $\omega_2 = \sqrt{\frac{k_2}{m_2}}$ is the local resonance frequency of the internal resonator.

If we consider the outer mass to be an effective mass, m_{eff}, we can derive an expression for the force acting on the effective mass of the system, using free body diagram, as

$$F_0 = -m_{eff}\,\omega^2 \hat{u}_1 \quad (6)$$

Comparing Eqs. (5) and (6) results in the effective mass, m_{eff}, in terms of the local resonance frequency of the structure, ω_2 and the input frequency, ω.

$$m_{eff} = m_1 + \frac{\omega_2^2 m_2}{\omega_2^2 - \omega^2} \quad (7)$$

From Eq. (7), if ω gets close to local resonance frequency ω_2, we get a vertical asymptote approaching positive infinity from one side and negative infinity from the other. It shows a region within a certain range of frequency, where the effective mass of the system becomes negative. This is also the region of formation for the frequency band gap.

Considering the stationary mass of the system as $m_{st} = m_1 + m_2$, we derive the normalized mass $\frac{m_{eff}}{m_{st}}$ as a function of the normalized operating frequency $\frac{\omega}{\omega_2}$. Simplifying Eq. (7), we obtain

$$\frac{m_{eff}}{m_{st}} = 1 + \frac{\theta}{1+\theta}\left[\frac{(\omega/\omega_2)^2}{1-(\omega/\omega_2)^2}\right] \quad (8)$$

Eq. (8) shows that the normalized mass depends on the normalized frequency ω/ω_2 and the term θ which is the ratio of the inner mass to the outer mass, $\theta = m_2/m_1$. The normalized parameter will approach negative values slightly positive of $\omega/\omega_2 = 1$.

3. Experimental methods

3.1. Materials selection

In the above section, it is shown that in locally resonant elastic metamaterials, the mass ratio θ, ratio of inner mass to outer mass, is a significant factor, whereby increasing mass ratio widens the frequency band gap region for negative effective mass. Therefore, it is beneficial to use a heavy material for the material selection of the inner mass. There are some metals which have very high density such as tungsten (density 19,250 kg/m^3) and lead (density 11,340 kg/m^3). However, they have poor machinability and pose health and safety concern. Therefore, in order to avoid such problems, the inner mass material chosen for this study is ultra machinable 12L14 carbon steel (density 7850 kg/m^3), and